Metaheuristic for School Bus Routing and Scheduling with Collaboration

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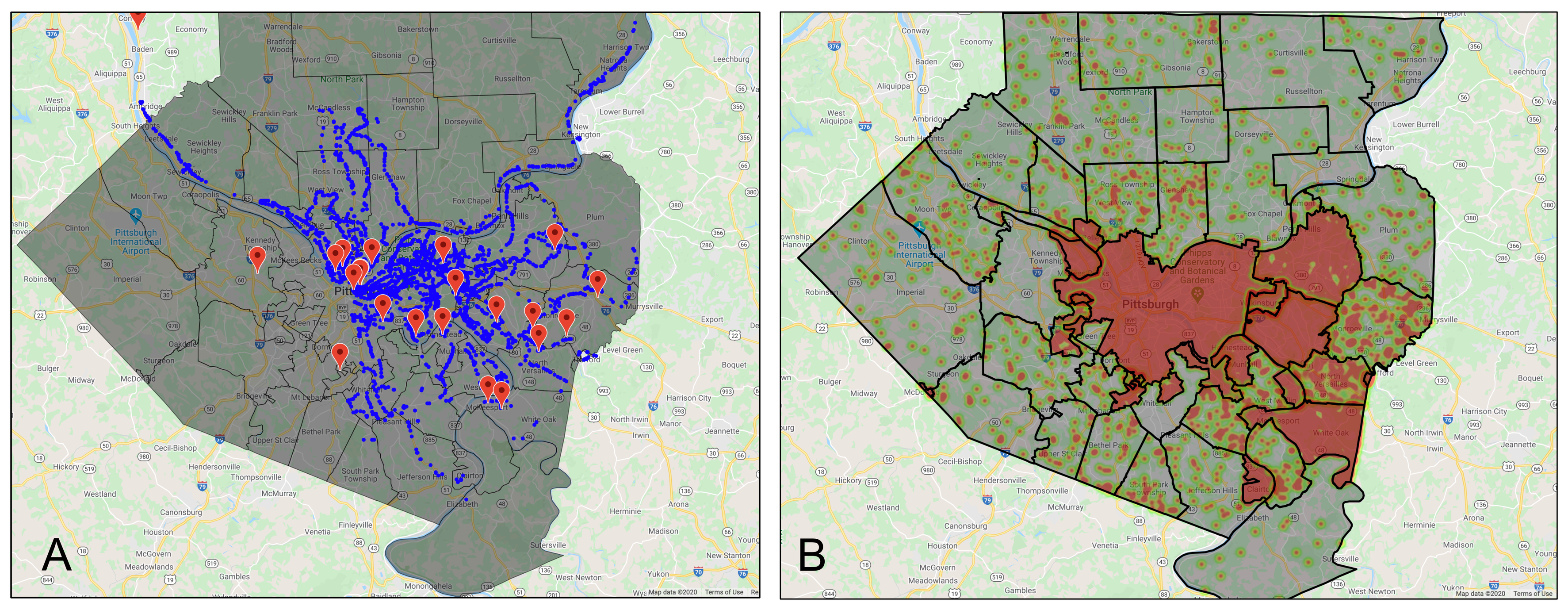
Introduction

The school bus routing problem (SBRP) is crucial because of its impact on economic and social objectives. To this end, this project considers the problem of multi-school SBRP with collaboration in which three simultaneous decisions have to be made for a multi-school transportation network: (**1**) select the subset of stops to visit amount all potential stops (**2**) assign each student to a stop (s)he should walk to (**3**) generate optimal routing and scheduling for each bus while allowing collaboration across multiple schools, so that the number of buses and the total travel distance are minimized. The scale and complexity of a multi-school SBRP is visualized in Fig. 1 with the example of Allegheny County school districts. The state-of-art methods are classified in Tab.1. In this project, subproblems (**1**) and (**2**) are solved using an exact mixed-integer programming (MIP) model; a two-stage metaheuristic method based on a tabu search is proposed for subproblem (**3**). The performance of these methods are evaluated by computational studies.

Reference	Stop Assignment	Bus Routing	Bus scheduling
[Bertsimas et al., 2019]	✓	✓	✓
[Spada et al., 2005]		✓	✓
[Schittekat et al., 2013]	✓	✓	
[Park et al., 2012]	✓	✓	✓
[Chen et al., 2015]		✓	✓
[Caceres et al., 2017]		✓	
[Laporte and Potvin,]		✓	

Tab. 1: State-of-the-art methods classified by the sub-problems studied.

Visualization



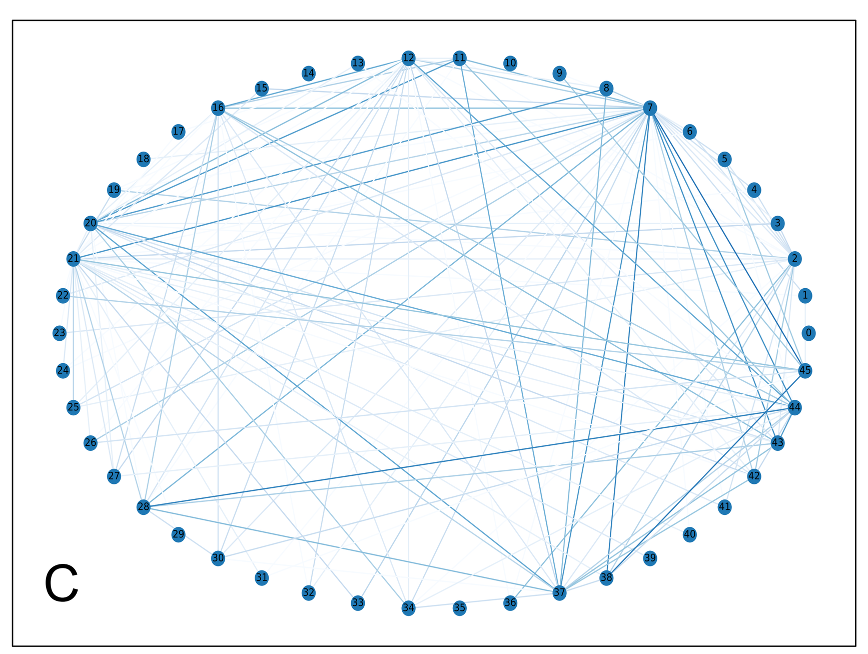


Fig. 1: Geographic and network visualization of the SBRP complexity for Allegheny County. (***A***) Allegheny County school districts 2018-2019 data, with blue nodes representing bus stops and red nodes representing schools. (***B***) Allegheny County student population heatmap. (***C***) Network of student flow between Allegheny County school districts during a morning service, with nodes representing school districts and intensity of the edges representing the flow size.

Stop Selection and Assignment

The problem of stop selection (select the stop for the fleet to visit) and assignment (assign each student to a stop) can be solved jointly as a single integer optimization problem with maximum walking distance constraint. Both (I) the number of stops selected and (II) total walking distance for students are minimized. The following MIP is solved for each school.

$$
\begin{aligned}
\min \quad & \sum_{c\in\mathcal{C}} z_c + \beta \sum\sum d_{p,c} y_{p,c} && && \text{(1a)}\\
\text{s.t.} \quad & y_{p,c} \le z_c && \forall p \in \mathcal{P}_S, c \in C_p && \text{(1b)}\\
& \sum_{c\in\mathcal{C}_p} y_{p,c} = 1 && \forall p \in \mathcal{P}_S && \text{(1c)}\\
& y_{p,c} \in \{0,1\} && \forall p \in \mathcal{P}_S, c \in C_p && \text{(1d)}\\
& z_c \in \{0,1\} && \forall c \in \mathcal{C} && \text{(1e)}
\end{aligned}
$$

The parameter β controls the trade-off between the two objectives. When β is larger than a threshold value, students will be assigned to the nearest stop to their home; for β value close to zero, students may walk to longer in order to consolidate several stops. This trade-off is explored on a synthetic instance (as shown in Fig. 2) with Pareto frontier shown in Fig. 3.

Simulation Study

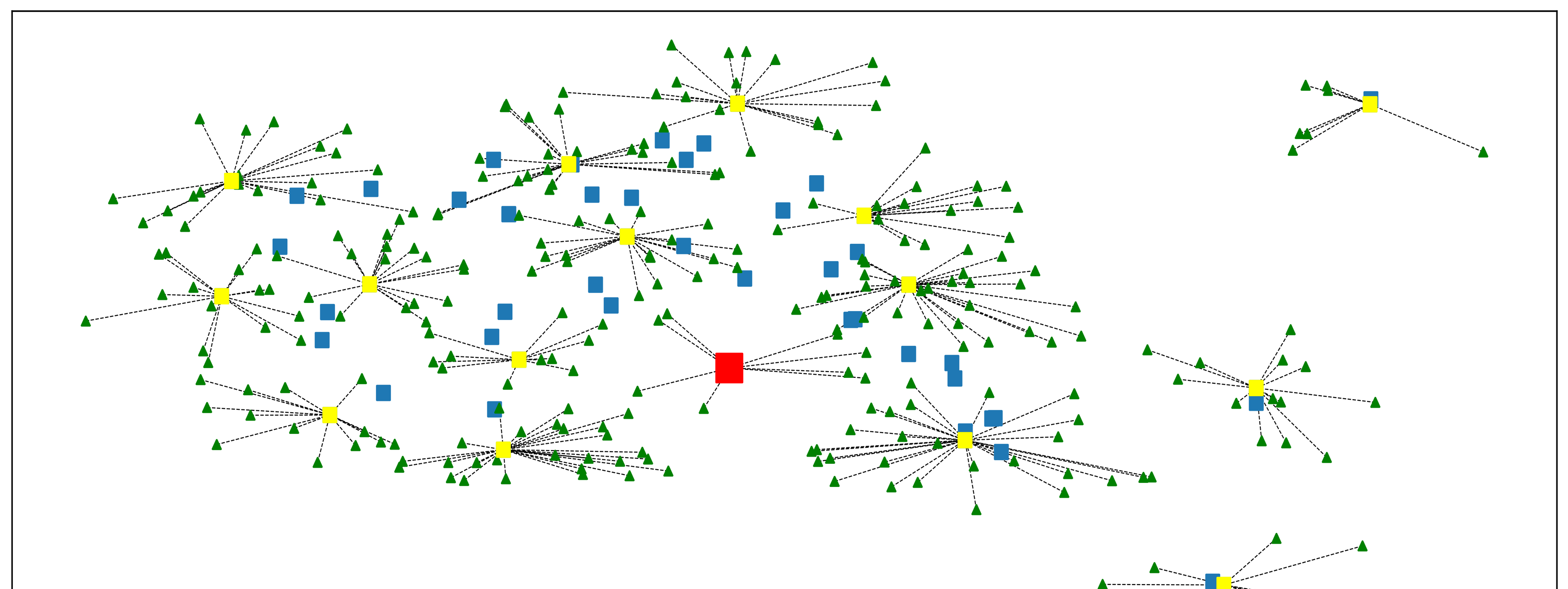


Fig. 2

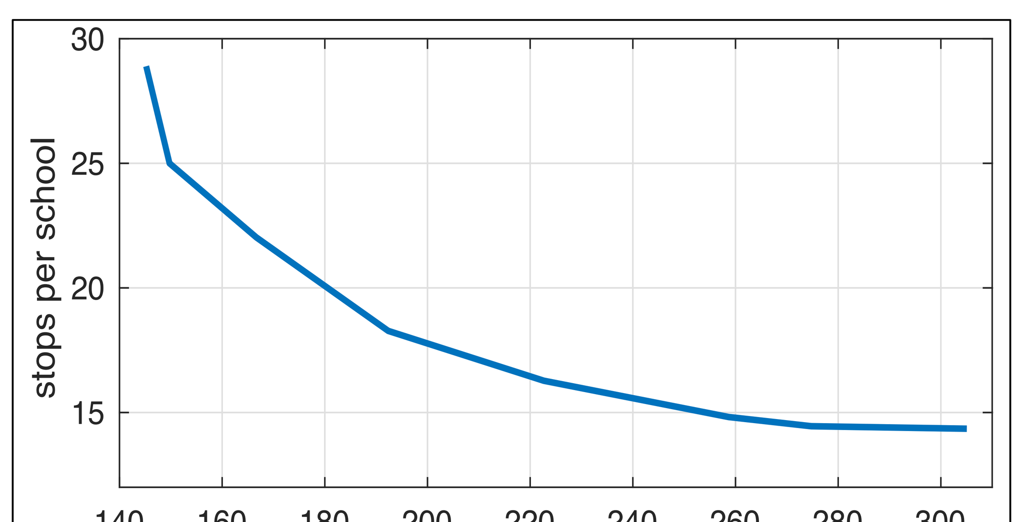


Fig. 3

Fig. 2: Stop selection and assignment result on a synthetic instance with a single school, 55 potential stops and 256 students. Green triangles: student homes; Red squares: school; Yellow squares: stops selected (15); Blue squares: stops not selected (40); Edges: stop assignment for each student.
Fig. 3: Trade-off between stops per school and average student walking distance. The simulation is performed on the instance shown in Fig. 2. The curve suggests diminishing returns in increasing the average student walking distance.

School Bus Routing and Scheduling with Bus Sharing

Given the solution to the stop selection and assignment, the last stage of the multi-school SBRP is to generate optimal routing and scheduling for each bus while allowing collaboration across multiple schools. Formally, this subproblem can be defined as a pickup and delivery problem with time windows (PDPTW). We proposed a two-stage metaheuristic method based on a tabu search. The method first finds a decentralized initial solution that is optimal for each school in isolation. This initial solution is then used to initialize the search that tries to find the centralized bus routing and scheduling solution that is optimal across all schools with collaboration.

Simulation Study

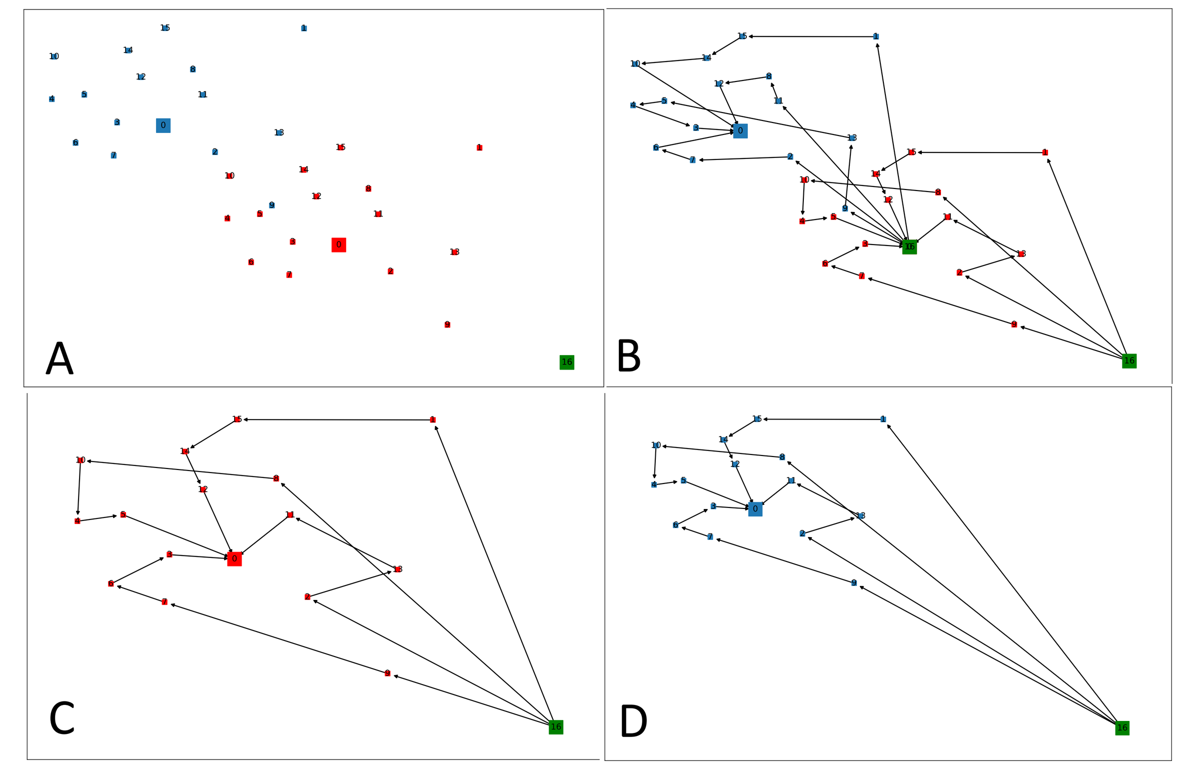


Fig. 4: Visualization of bus routing and scheduling problem. Demand at each stop is generated by solving the stop selection and assignment problem. (***A***) Synthetic data. Green node: bus depot; Red nodes: school I (large) and stops with demand for school I (small); Blue nodes: school II (large) and stops with demand for school II (small). (***B***) Centralized solution with collaboration. (***C***) (***D***) Decentralized solution for school I and school II.

The proposed method is tested on synthetic data. Fig. 4 shows both the decentralized (Fig. 4 (C) (D)) and the centralized solutions (Fig. 4 (B)) to an instance of two school system with a total of 256 students and a bus capacity of 70. Furthermore, simulations are performed on four different instances generated by varying the geographical information. Two performance metrics (I) number of buses and (II) total distance are compared between the decentralized solution and the centralized solution. As shown in Tab. 2 centralized method with collaboration improves both metrics significantly (in particular, a [22.4%, 26.8%] improvement on total distance).

Instance	N_{sch}	N_{stops}	N_{stud}	#Buses (Decentralized)	#Buses (Centralized)	Total Distance (Decentralized)	Total Distance (Centralized)
1	2	32	502	8	5	12,096m	9,381m (22.4%)
2	2	32	502	8	6	13,125m	10,212m (22.2%)
3	2	32	502	8	5	11,512m	8,725m (24.2%)
4	2	32	502	8	4	10,879m	7,962m (26.8%)

Tab. 2: Comparison between decentralized and centralized routing and scheduling solutions on synthetic instances

Future Work

For future work, the performance of the methods should be evaluated on large-scale data sets and compared with the state-of-the-art. In addition, a mixed-load post-improvement algorithm of the current routing and scheduling method can further improve the objectives.