Myopically Verifiable Probabilistic Certificate for Long-term Safety

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Control & Learning group @ CMU

理学

Stochastic safe control
Robust control
Optimization
Information theory …

科学

Neuroscience
Biomolecular control…

工学

Today’s talk

理学

Stochastic safe control
Robust control
Optimization
Information theory …

工学

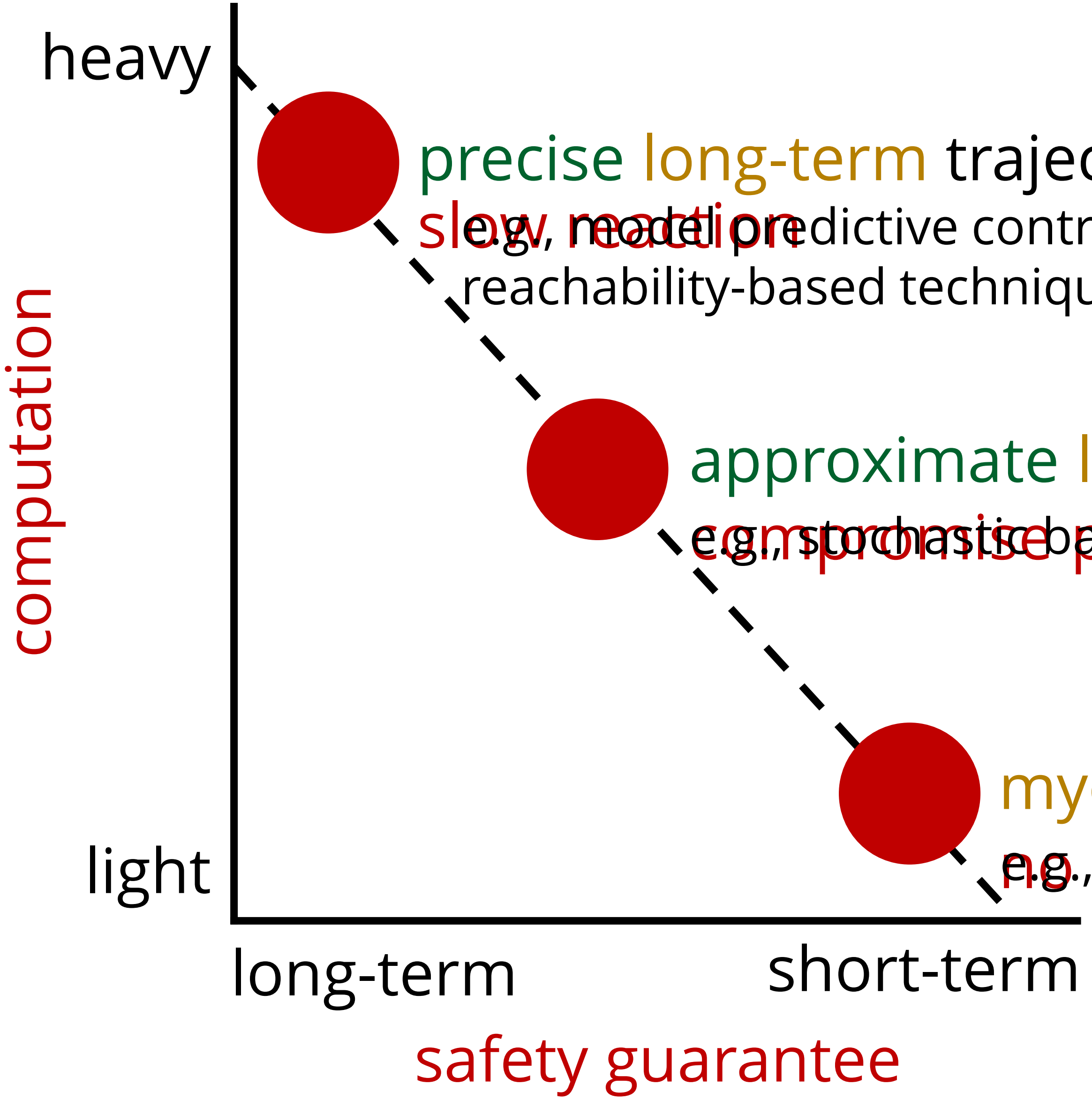
Safety is critical for intelligent systems

Autonomous vehicles

Cobots
Intelligent manufacturing

Drones

Real-time safety certificate in uncertainty environment



precise long-term trajectories

slow reaction

e.g., model predictive control (MPC), constrained optimization safety, reachability-based techniques, conditional value-at-risk (CVaR) [1-4]

approximate long-term safe probability

compromise performance

e.g., stochastic barrier certificate [5]

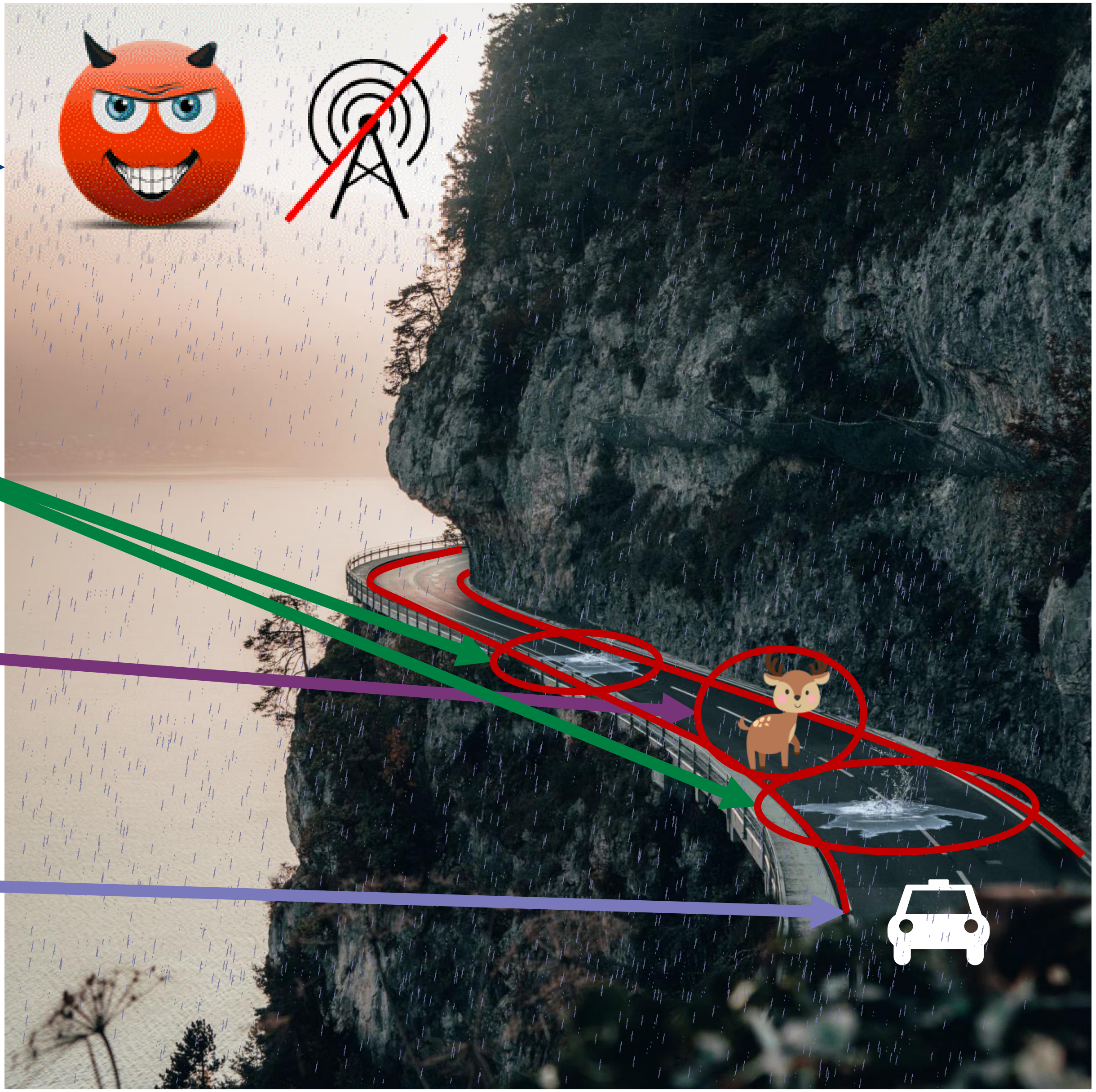
myopic safe probability

no long-term guarantee

e.g., barrier function-based control [6-7]

Safety is critical for intelligent systems

- Resilience
- Adaptability
- Collision Avoidance
- Regular Operation

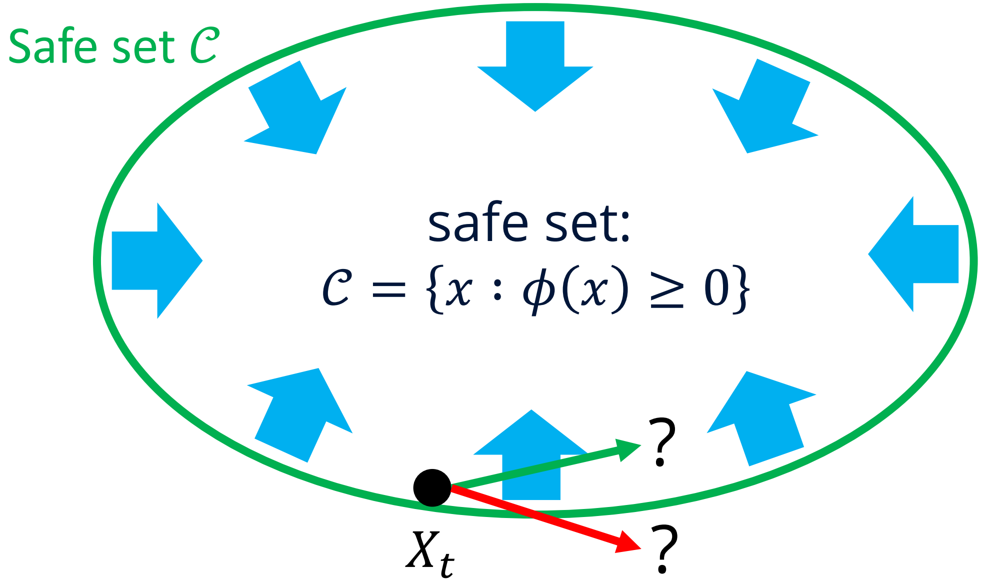


Safety is critical for intelligent systems

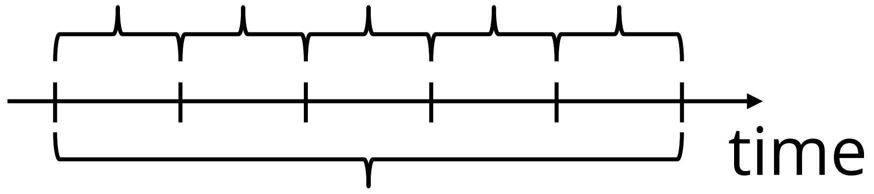
Challenges: achieving safety in uncertainty

Existing approach:
Control barrier function…

safe at next time => safe at all time ✗



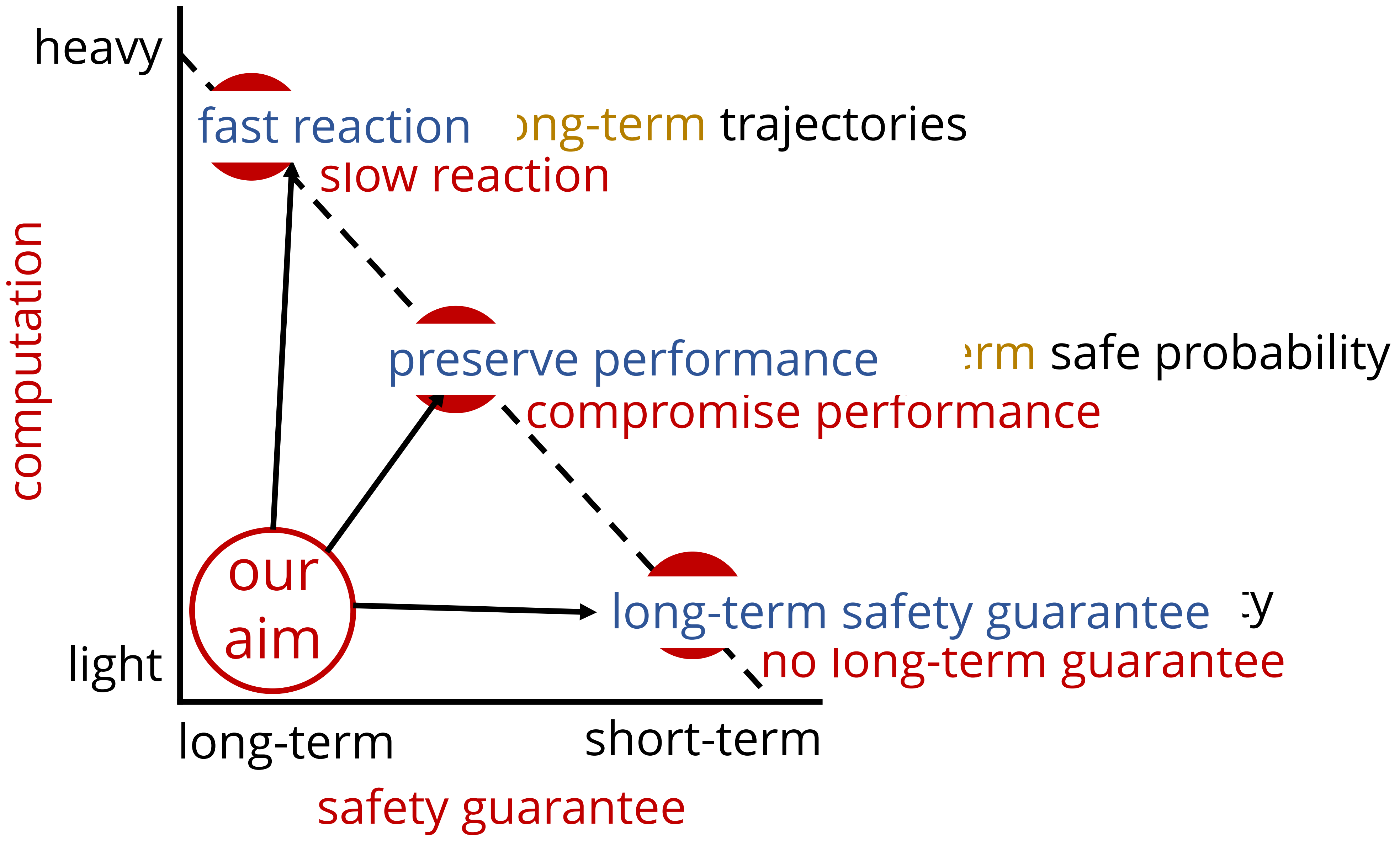
safe with probability
$1 - \delta$ at each step



unsafe with high probability
in a long term

Under stochastic
uncertainties

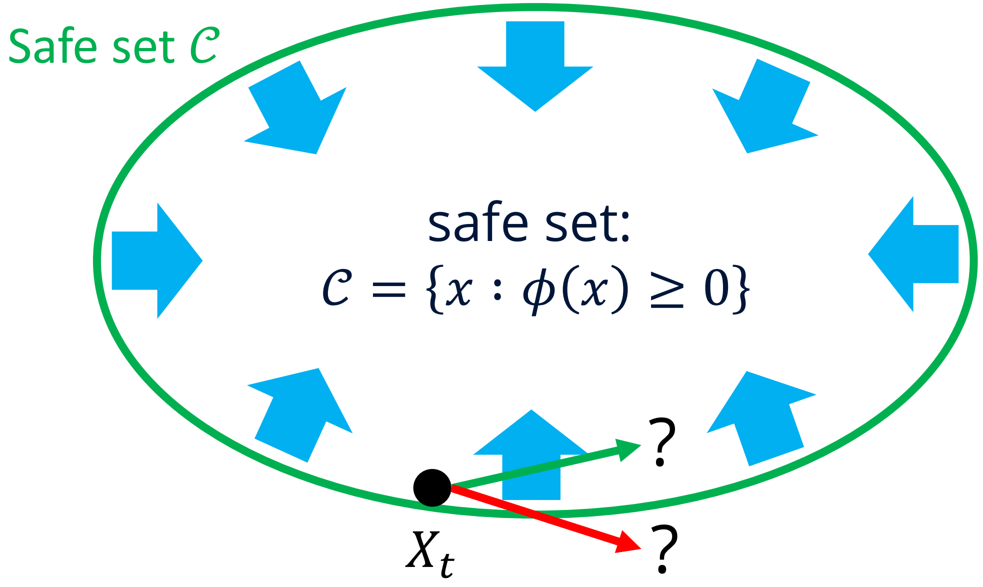
Aims



heavy
light
computation
fast reaction
ong-term trajectories
slow reaction
preserve performance
erm safe probability
compromise performance
our
aim
long-term safety guarantee
y
no long-term guarantee
long-term
short-term
safety guarantee

Proposed Method: Intuitions

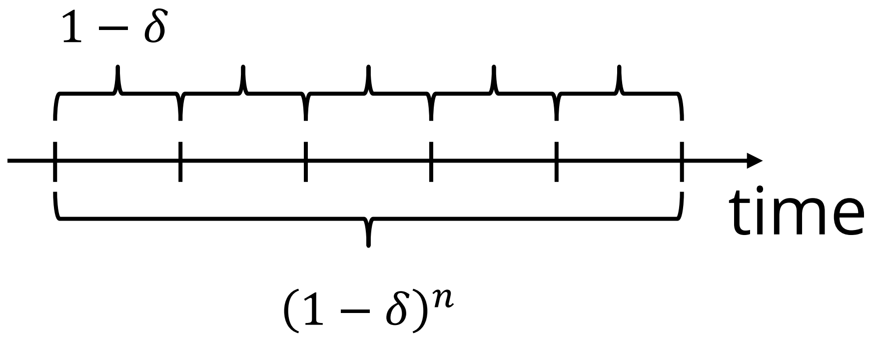
Existing approach:
Control barrier function…



Under stochastic uncertainties

safe at next time => safe at all time ✗

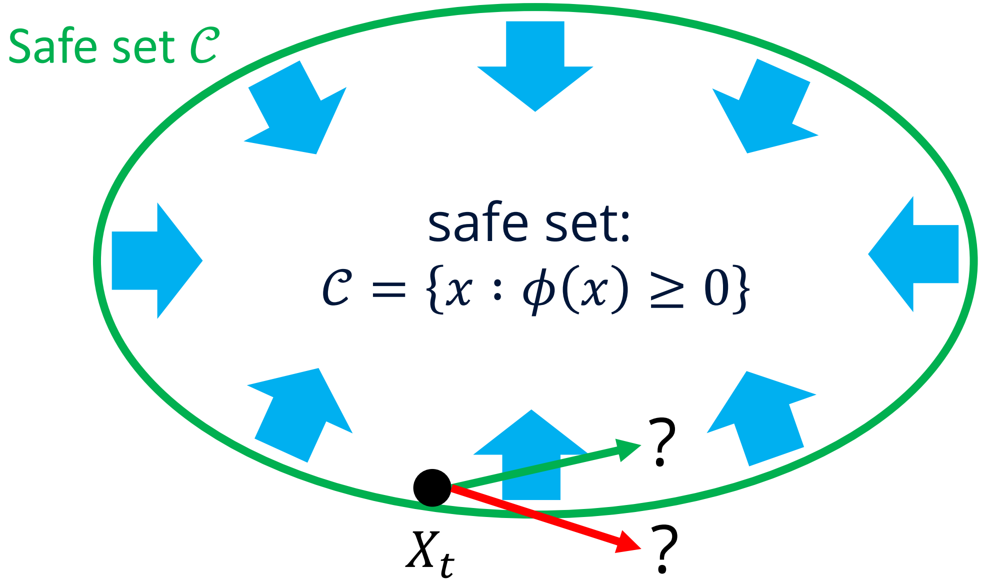
safe with probability $1 - \delta$ at each step



unsafe with high probability in a long term

Proposed Method: Intuitions

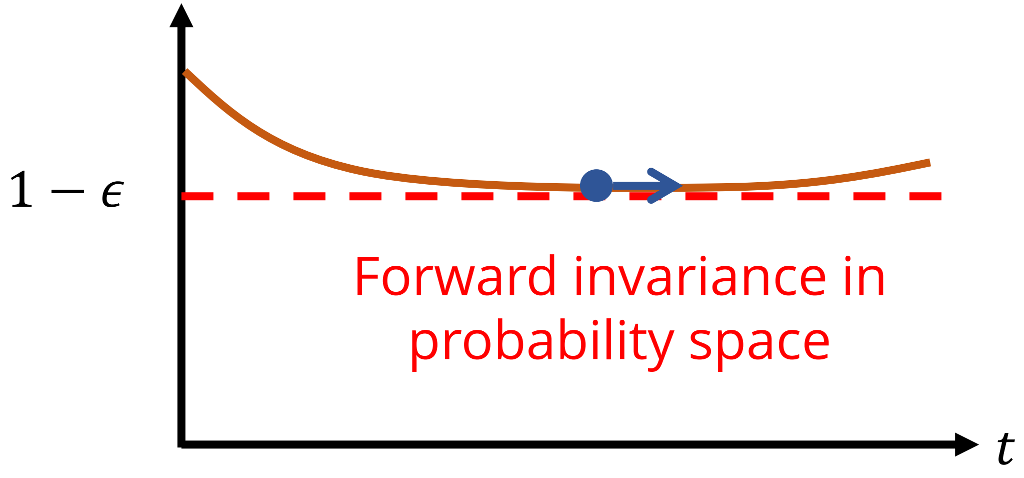
Existing approach:
Control barrier function…



Under stochastic uncertainties

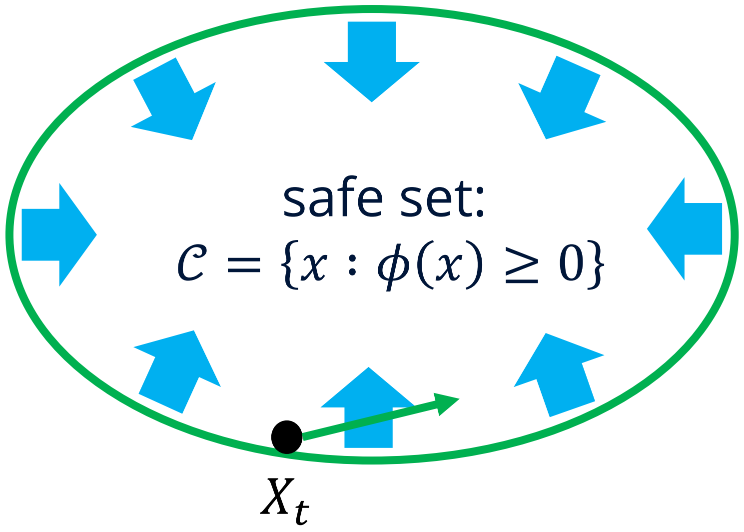
Proposed approach:

Long-term safety probability
$\boldsymbol{F}(X_t) = \Pr(X_\tau \in \mathcal{C}, \tau \in [t, t+T] | X_t)$



Proposed Method: Intuition

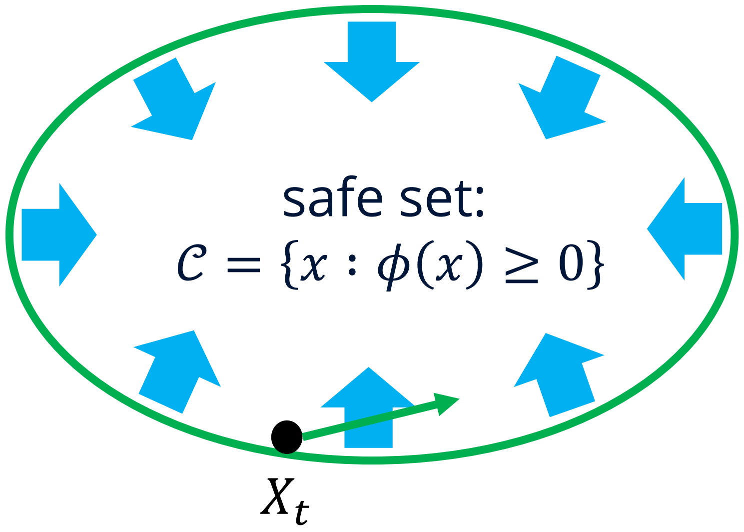
Control barrier functions:



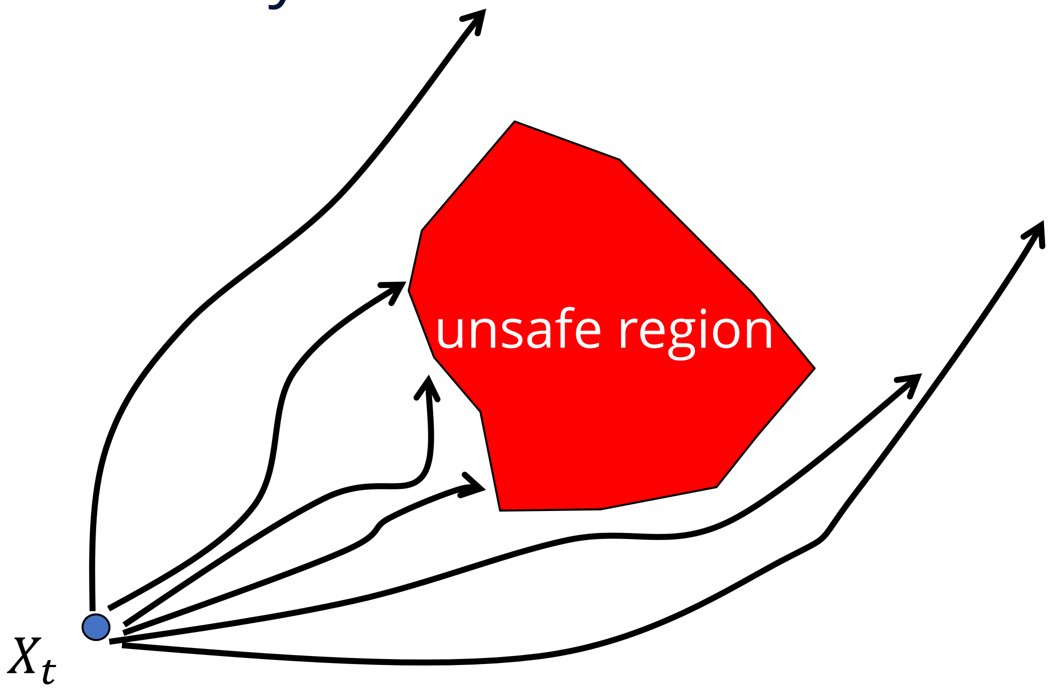
Reachability:

Proposed Method: Intuition

Control barrier functions:



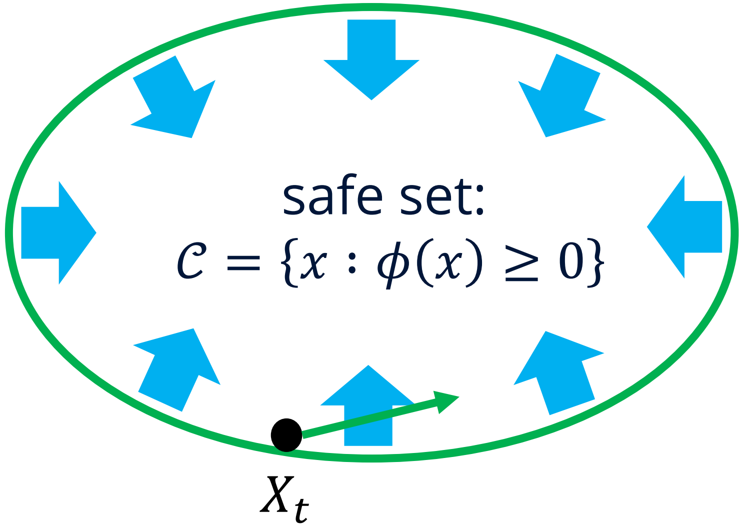
Reachability:



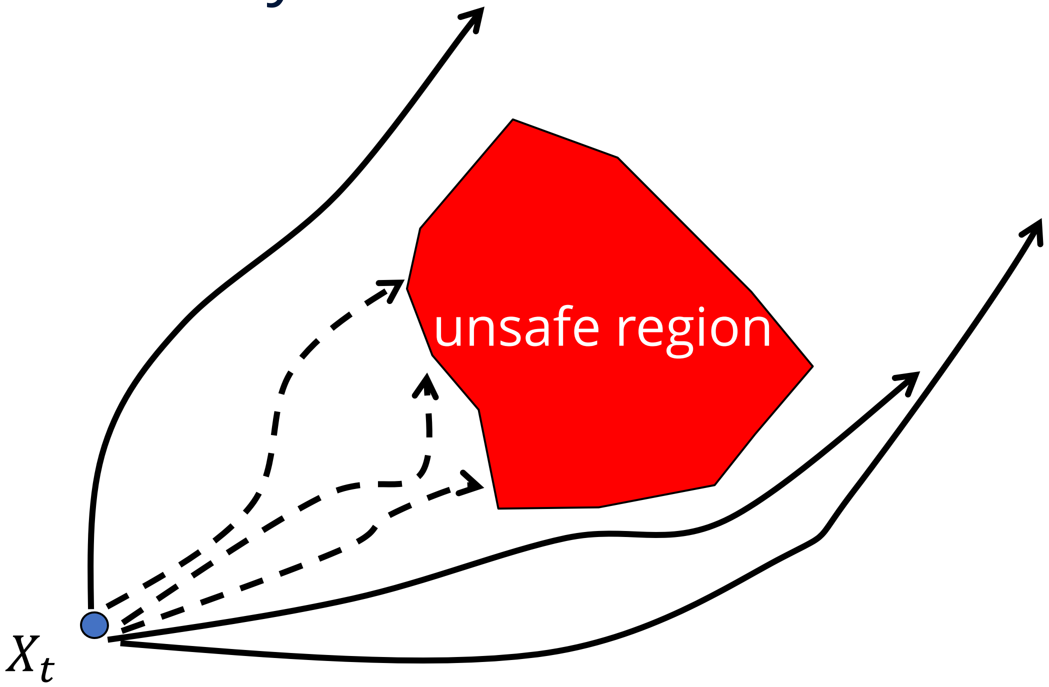
Forward rollout trajectories

Proposed Method: Intuition

Control barrier functions:



Reachability:



Forward rollout trajectories

Proposed Method: Intuition

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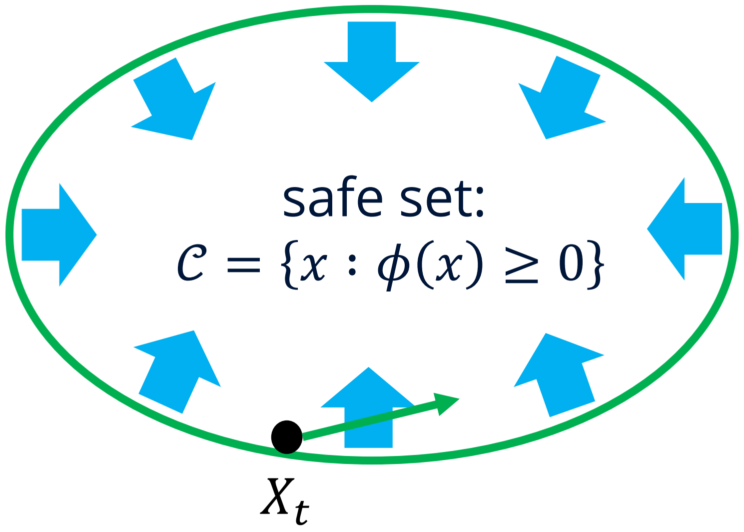
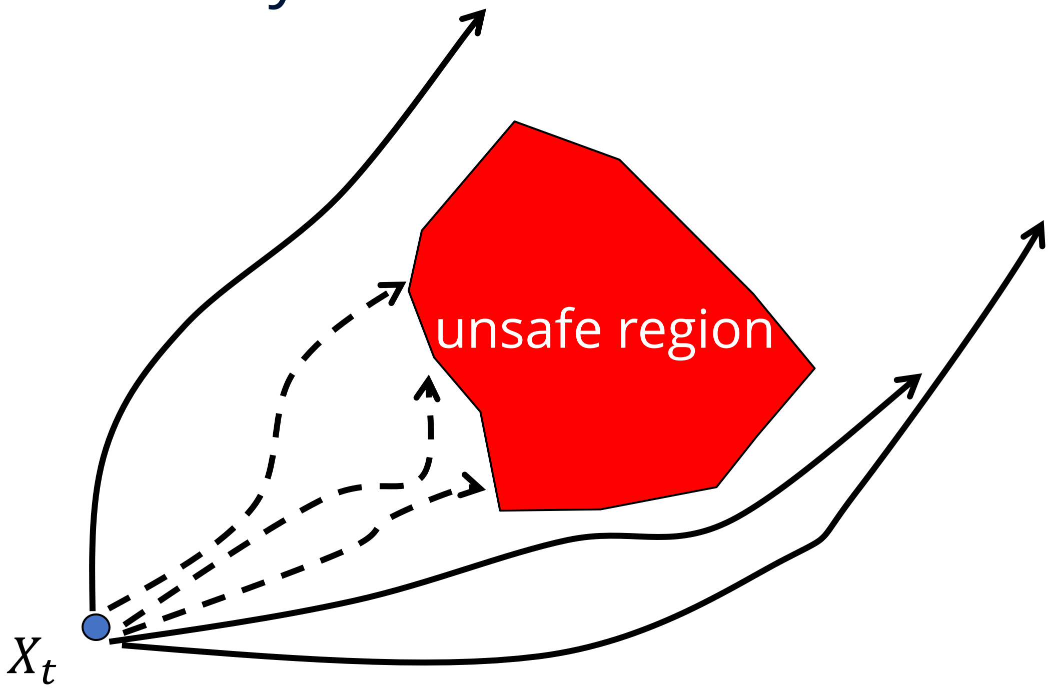


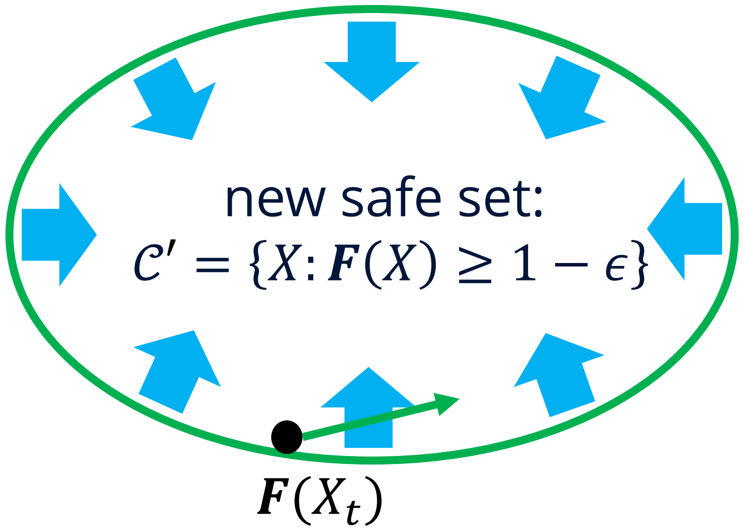
Reachability:



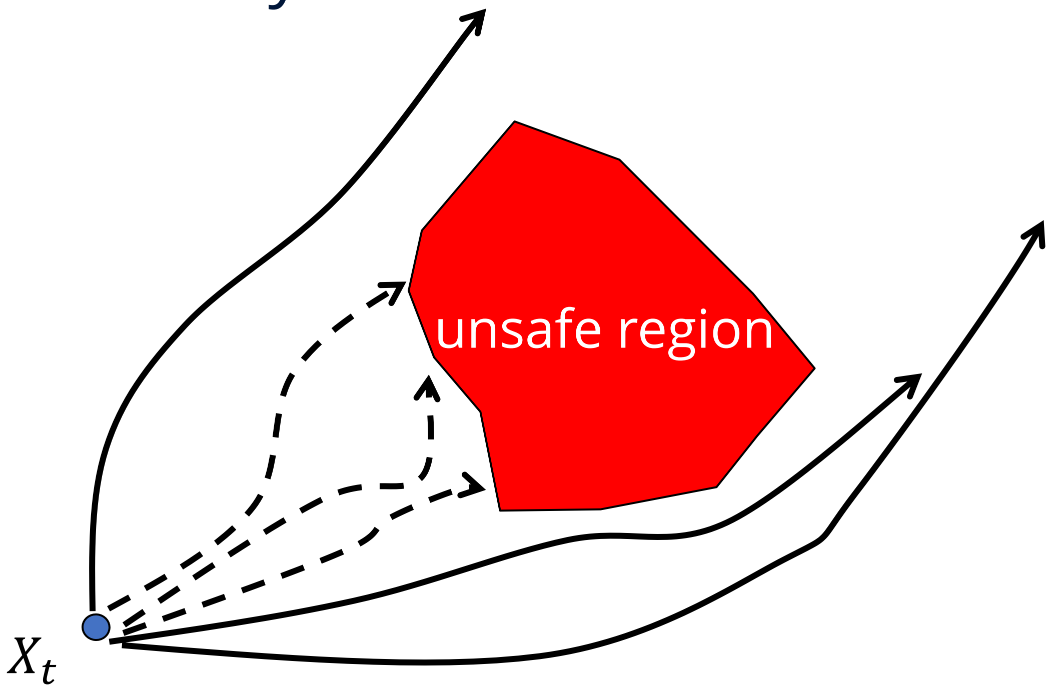
Encoded safety probability $\boldsymbol{F}(X_t)$ ⬅ Forward rollout trajectories

Proposed Method: Intuition

Control barrier functions:



Reachability:



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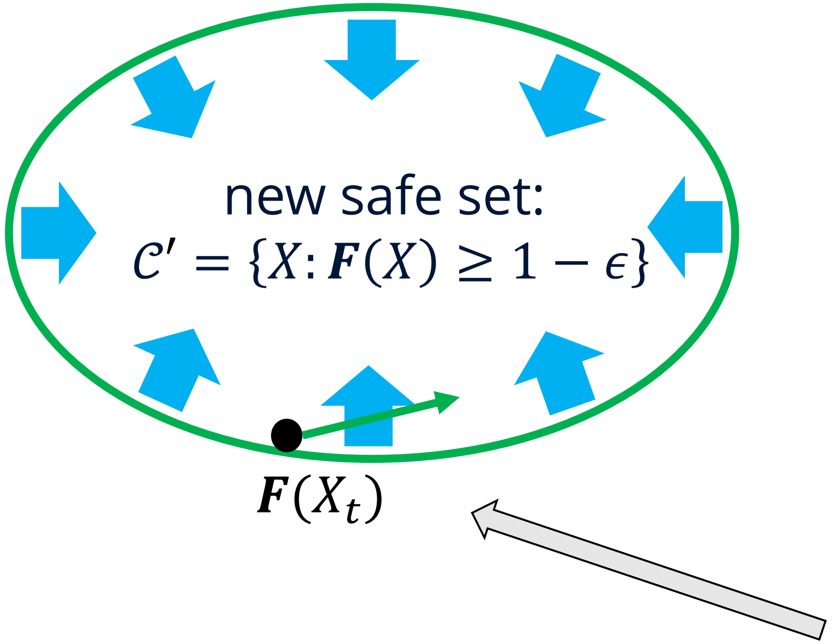
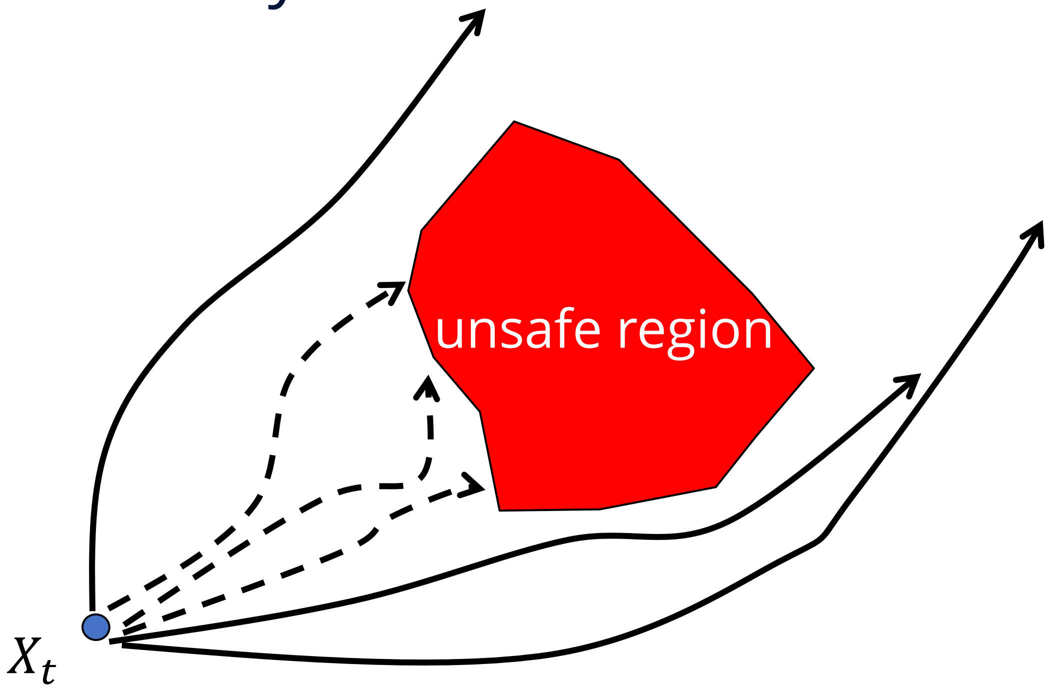


Reachability:

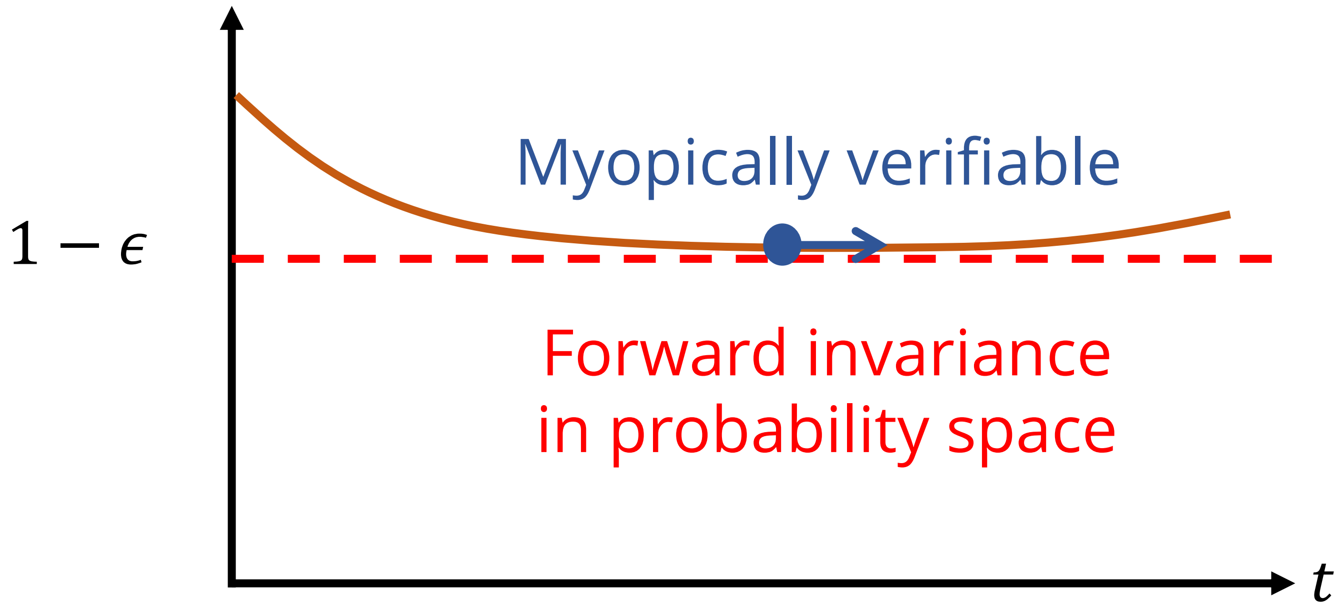


Encoded safety probability $\boldsymbol{F}(X_t)$ ← Forward rollout trajectories

Proposed Method

Long-term safe probability

$\boldsymbol{F}(X_t) = \Pr(X_\tau \in \mathcal{C}, \tau \in [t, t+T] | X_t)$



Proposed Safety Condition:

$$A\boldsymbol{F}(X_t) \geq -\alpha(\boldsymbol{F}(X_t) - (1 - \epsilon))$$

time derivative of safety probability

desired safety probability

A: infinitesimal generator

$\alpha: \mathbb{R} \rightarrow \mathbb{R}$ monotonically increasing, concave, $\alpha(0) \leq 0$.

Theoretical Guarantees

Theorem: Given

$$\boldsymbol{F}(X_0) > 1 - \epsilon,$$

if we choose the control action to satisfy

$$A\boldsymbol{F}(X_t) \geq -\alpha(\boldsymbol{F}(X_t) - (1 - \epsilon)) \text{ for } t > 0,$$

then we have

$$\Pr(X_\tau \in \mathcal{C}, \tau \in [t, t + T]) \geq 1 - \epsilon \text{ for } \forall t > 0$$

$\alpha: \mathbb{R} \to \mathbb{R}$ is a monotonically increasing concave function that satisfies $\alpha(0) \leq 0$.

Proposed Safety Condition

$$\boldsymbol{F}(X_t) = \Pr(X_\tau \in \mathcal{C}, \tau \in [t, t+T] | X_t)$$

$$A\boldsymbol{F}(X_t) \geq -\alpha(\boldsymbol{F}(X_t) - (1-\epsilon))$$

linear with respect to u

$$A\boldsymbol{F}(X_t) = \mathcal{L}_f \boldsymbol{F}(X_t) + \left(\mathcal{L}_g \boldsymbol{F}(X_t)\right)u + \frac{1}{2}\mathrm{tr}([\sigma(X_t)]^\intercal \mathrm{Hess}\boldsymbol{F}(X_t)[\sigma(X_t)])$$

constant given system dynamics

$$dX_t = (f(X_t) + g(X_t)U_t)dt + \sigma(X_t)dW$$

Simulation

system dynamic: $dx_t = (2x_t + 2.5u_t)\,dt + 2dw_t$

initial state: $x_0 = 3$

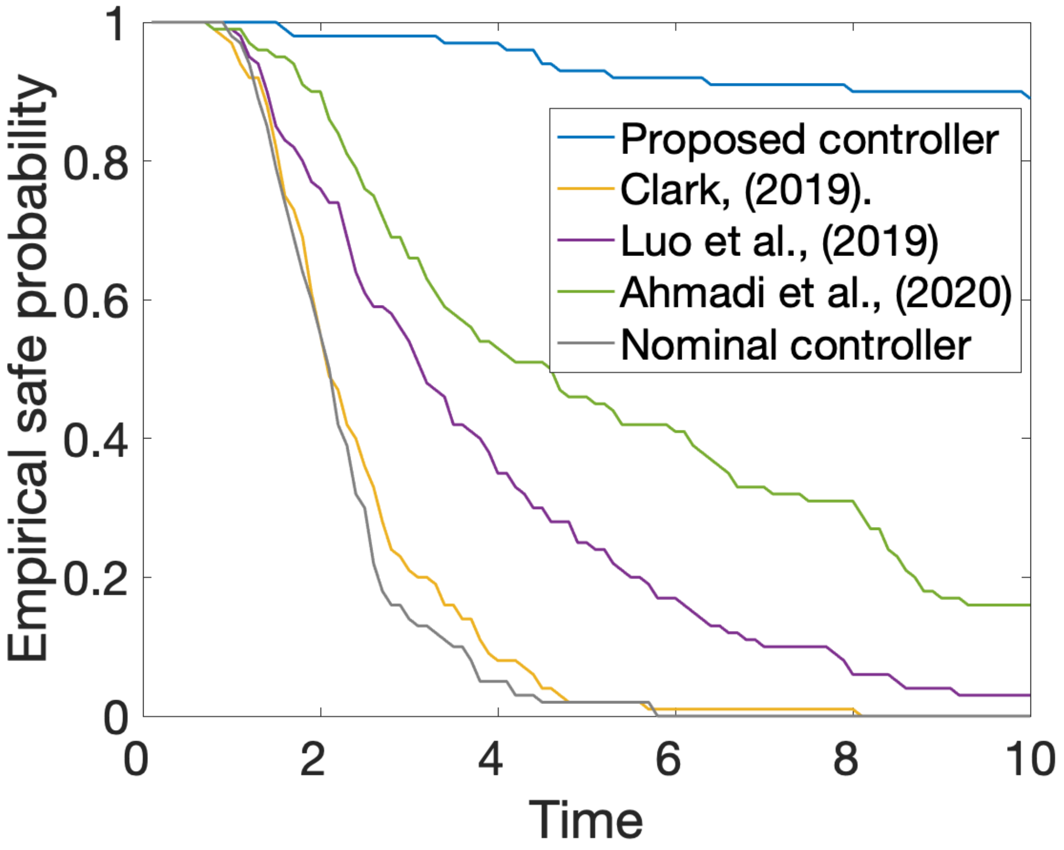
safe set: $\mathcal{C} = \{x \in \mathbb{R} : x - 1 > 0\}$

nominal controller: $N(x_t) = 2.5x_t$

desired safety probability: $1 - \epsilon = 0.9$

Simulation

Empirical safety probability:



Safety conditions:

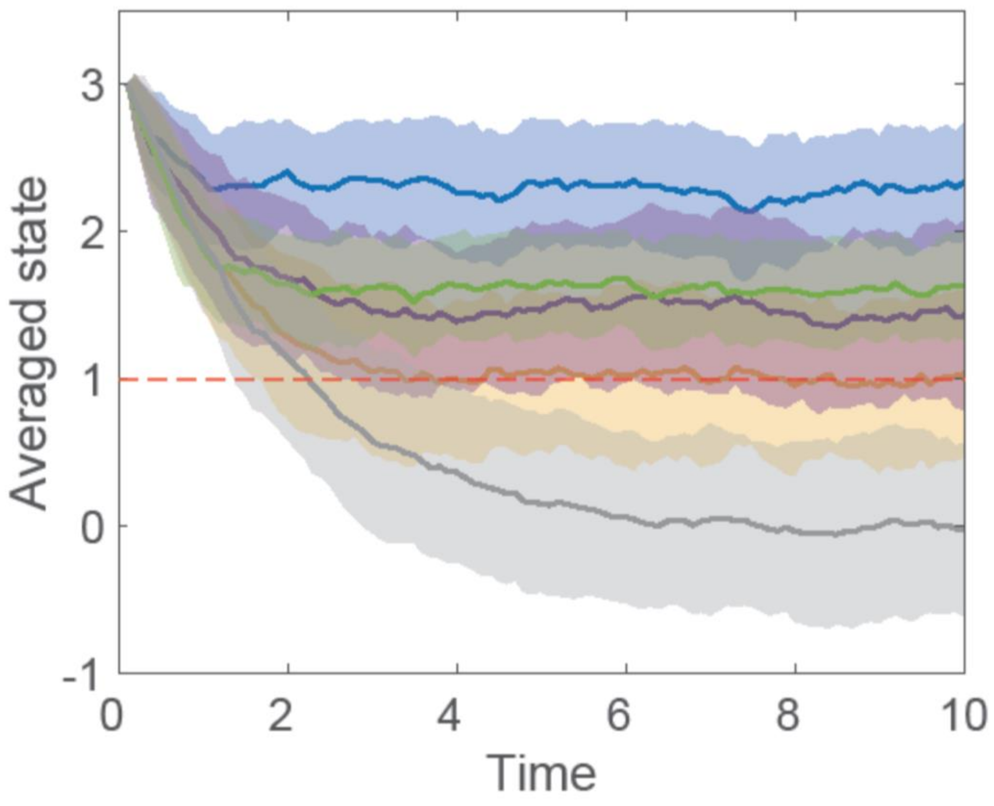
Proposed: $A\boldsymbol{F}(X_t) \geq -\alpha(\boldsymbol{F}(X_t) - (1-\epsilon))$

Clark: $A\phi(X_t) \geq -\alpha\phi(X_t)$

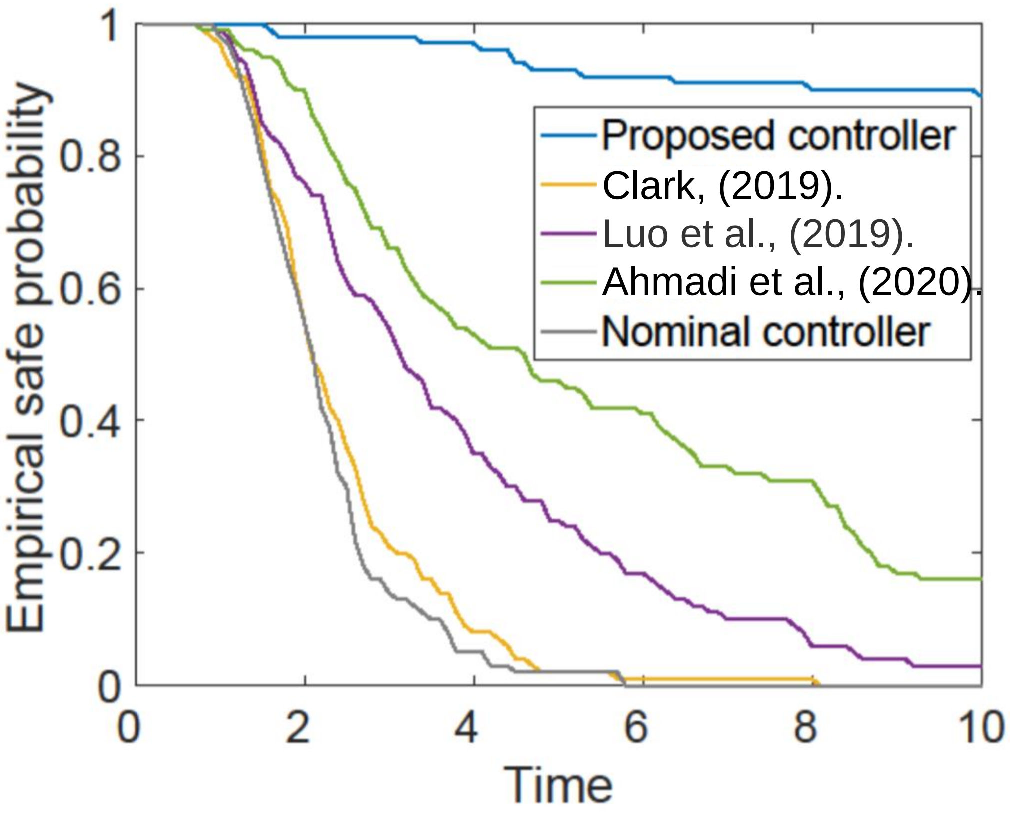
Luo et al.: $\mathbb{P}(d\phi(X_t, U_t) + \alpha\phi(X_t) \geq 0) \geq 1 - \epsilon$

Ahmadi et al.: $\mathrm{CVaR}_\beta(\phi(X_{t+1})) \geq \gamma\phi(X_t)$

Simulation



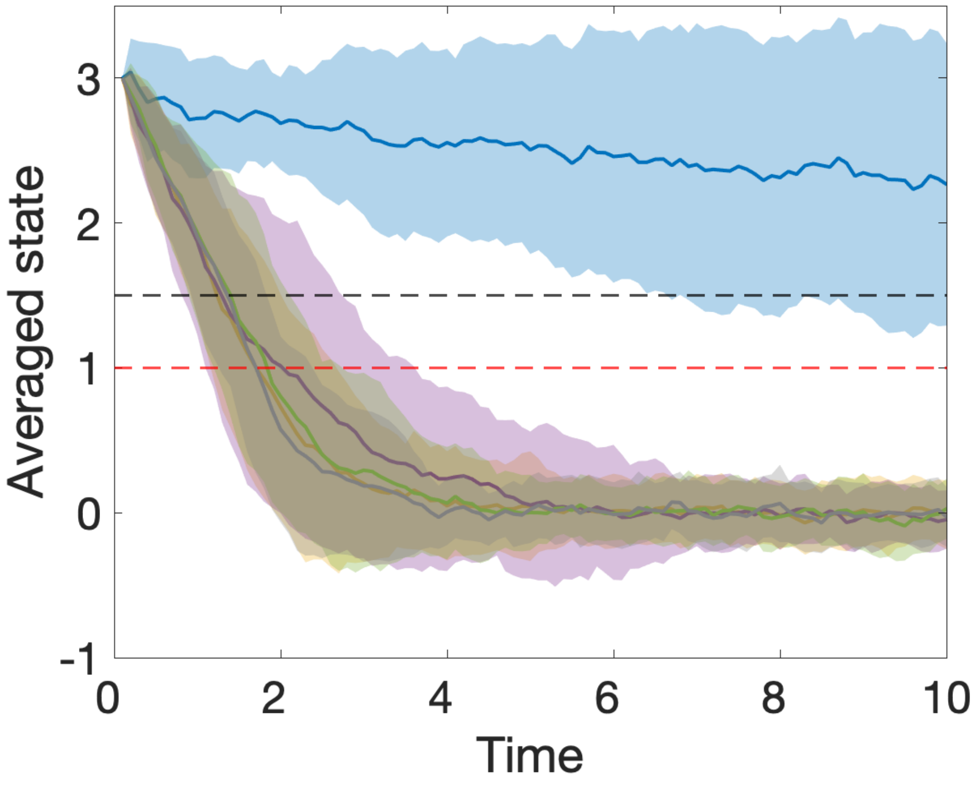
Averaged state
Time

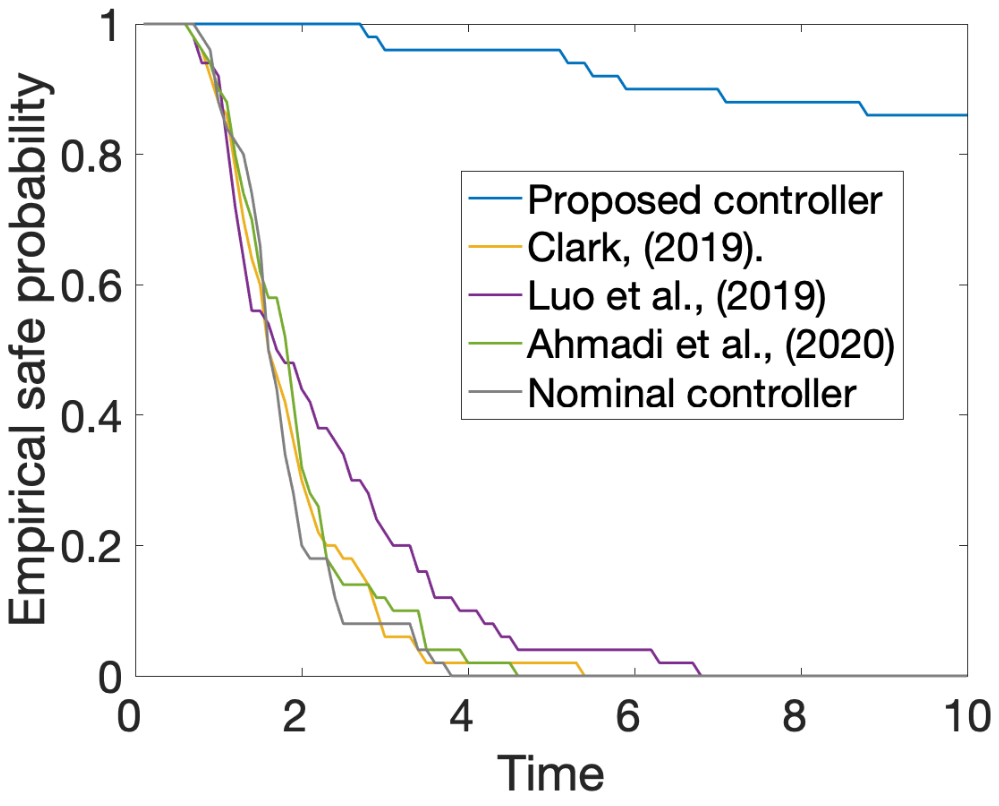


Empirical safe probability
Time
Proposed controller
Clark, (2019).
Luo et al., (2019).
Ahmadi et al., (2020).
Nominal controller

Simulation – Nonlinear trap

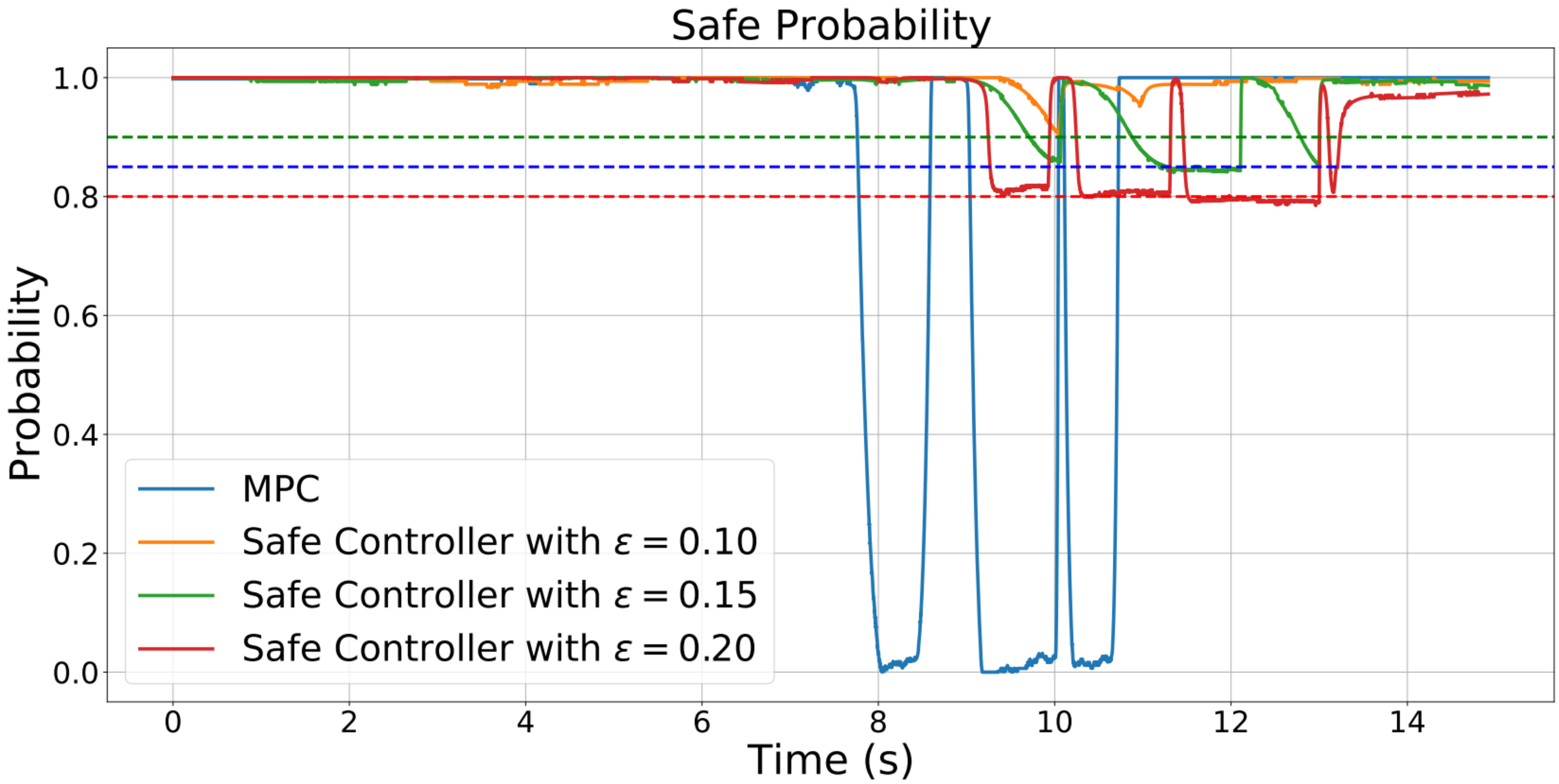
System becomes uncontrollable once reach state $x = 1.5$



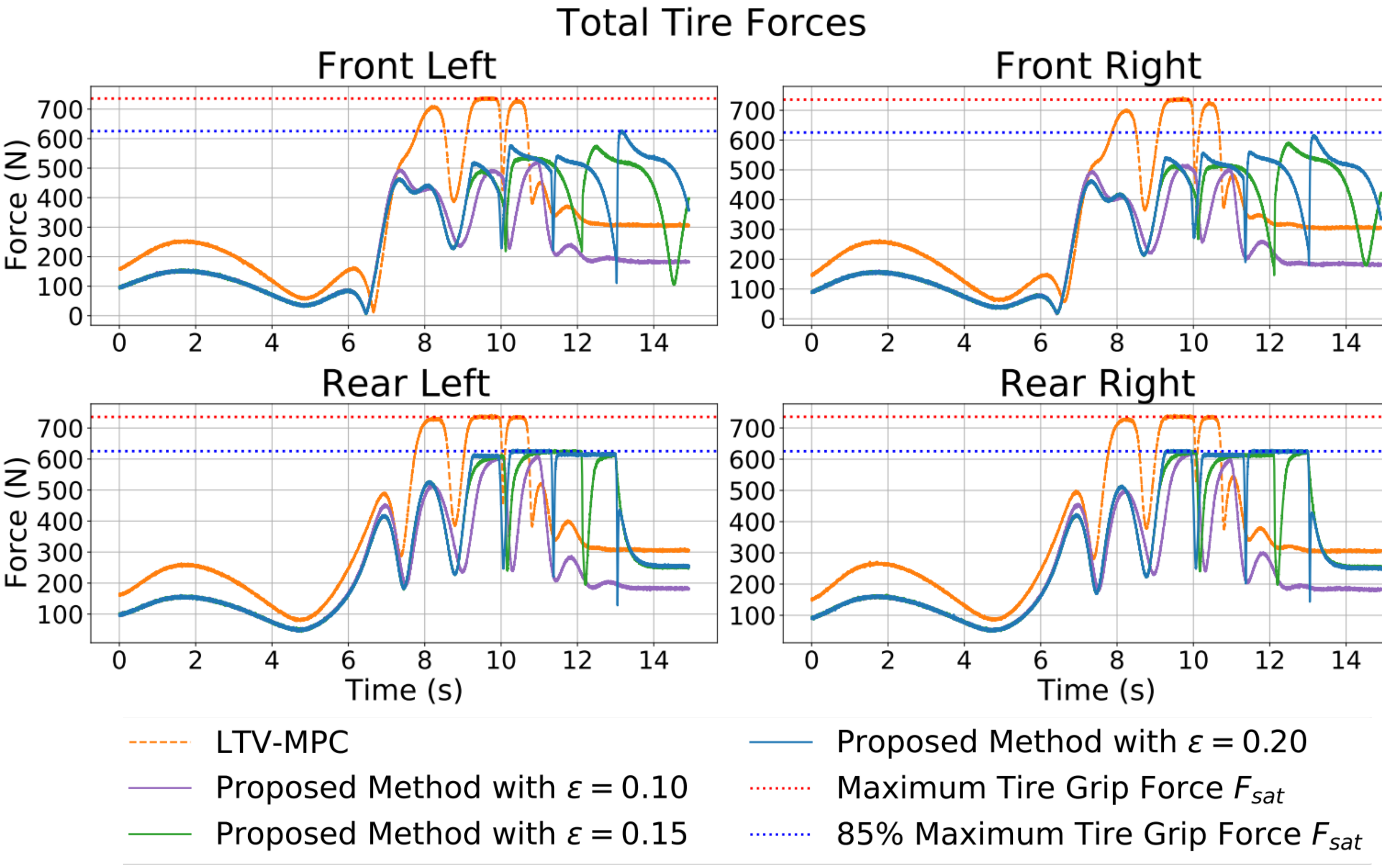


Setting: vehicle slippery -> lose of control -> loss of safety

Advantage 1: Long-term Safety Guarantee

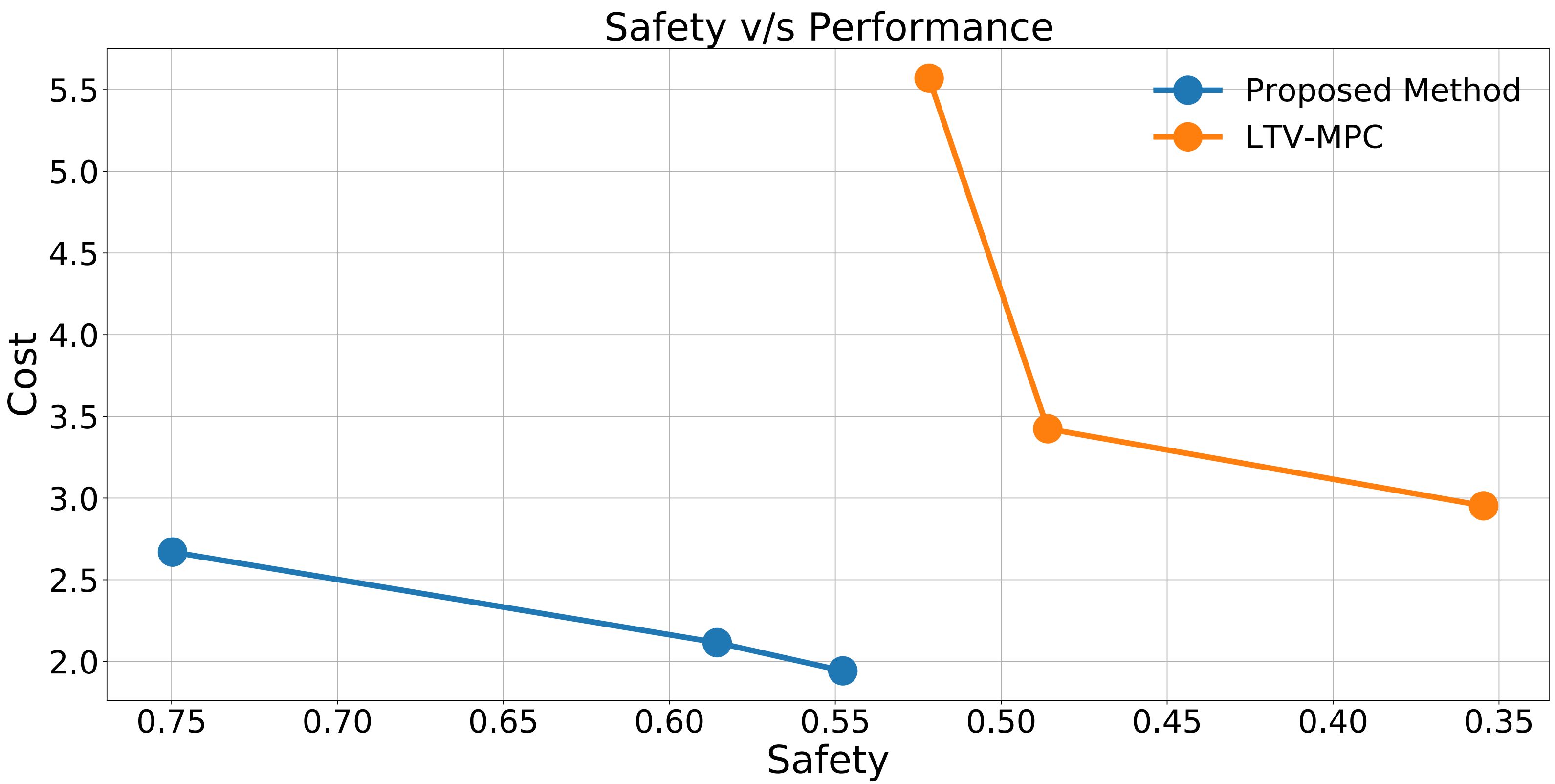


Advantage 1: Long-term Safety Guarantee (Cont'd)



Advantage 2: Better Performance Tradeoffs

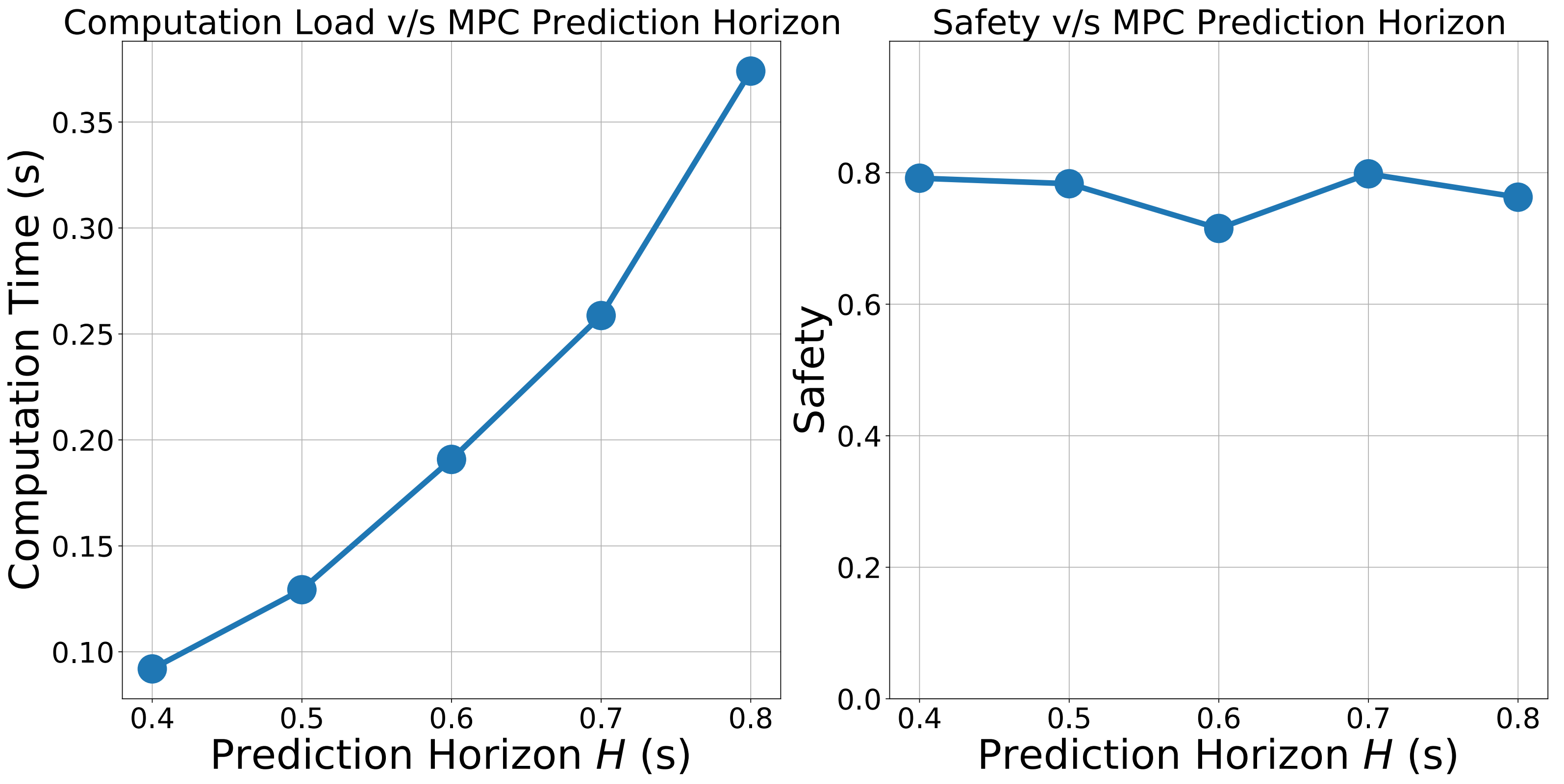
cost:
deviation from the reference trajectory



safety: satisfaction of the tire force limits

Advantage 3: Less Computation Costs

- Computation of MPC grows in $O(H^3)$
- Safety will not be compromised even with short outlook horizons



Today's talk

理学

Stochastic safe control
Robust control
Optimization
Information theory …

工学

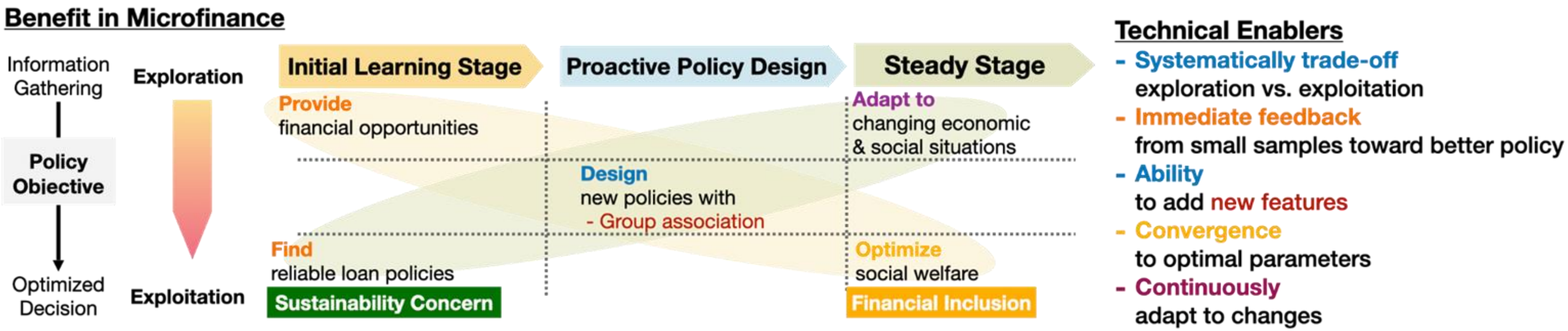
Microfinance from a control perspective

- Microfinance in developing areas has been proven to improve the local economy significantly.

However, building reliable microfinance system is challenging

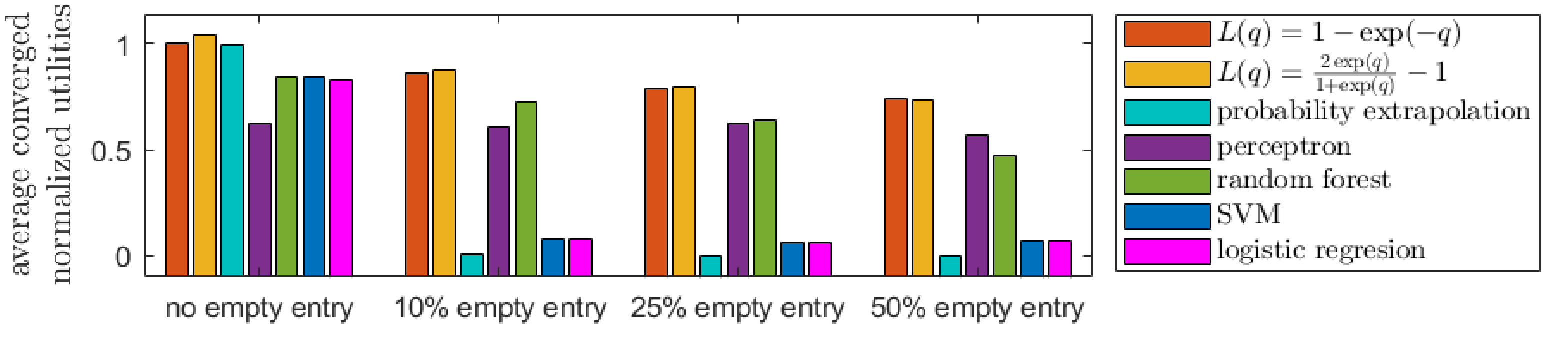
1. Complexity in understanding default process
2. Asymmetry, heterogeneity, and incomplete information of individual applications
3. The scarcity of available past data
4. The dynamically evolving social and economic conditions

Features of Proposed Algorithm:

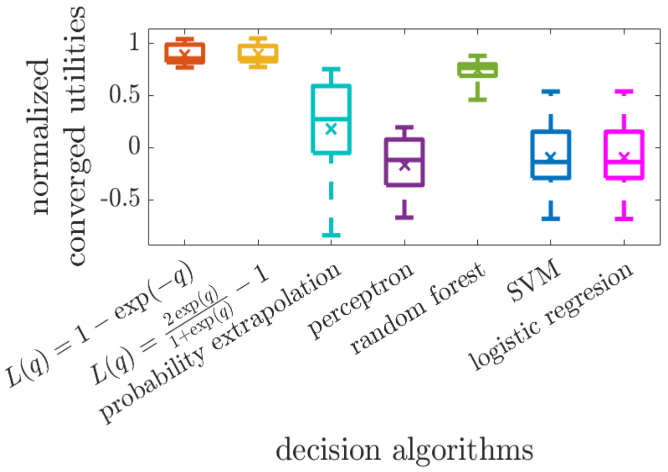


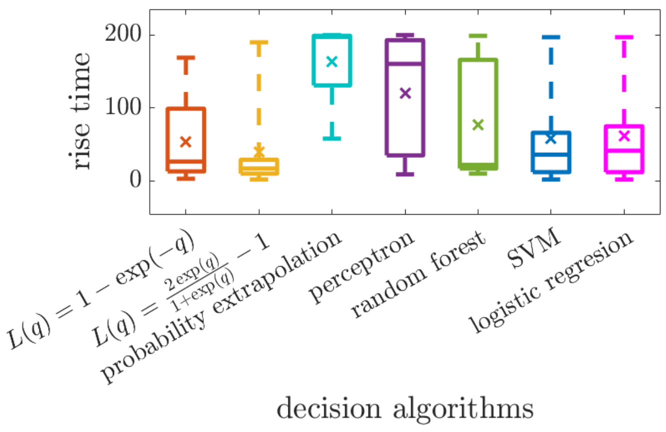
Microfinance from a control perspective

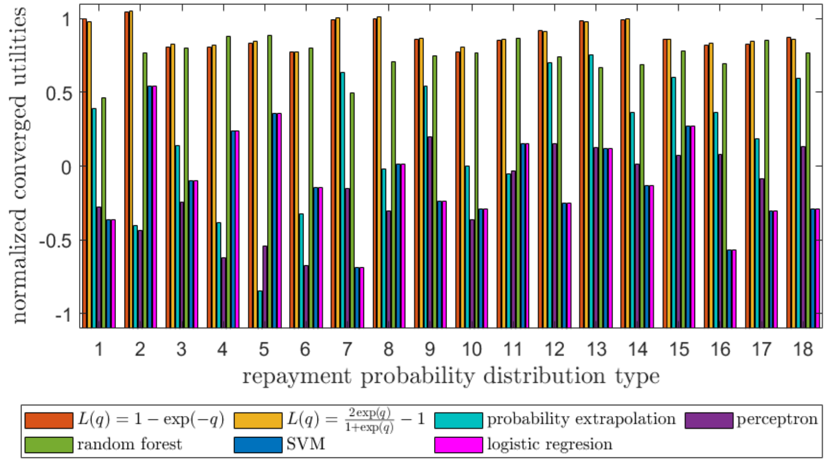
1. **Robustness against missing data**



2. **Ability to deal with diverse microfinance distributions**

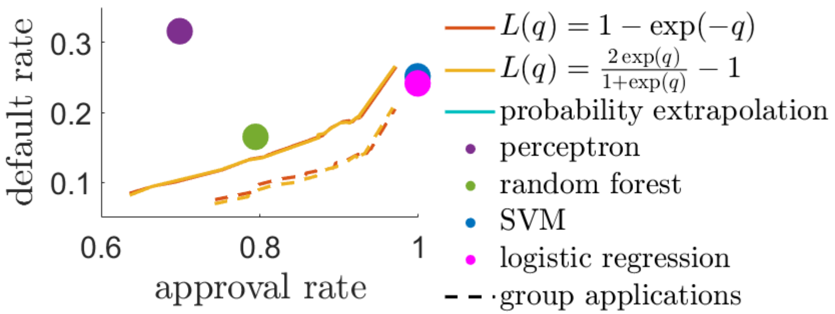




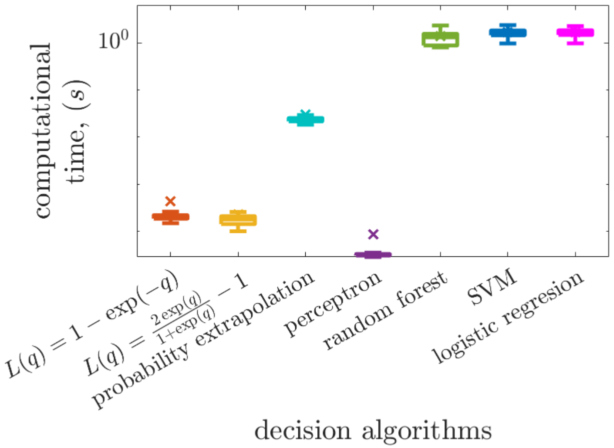


Microfinance from a control perspective

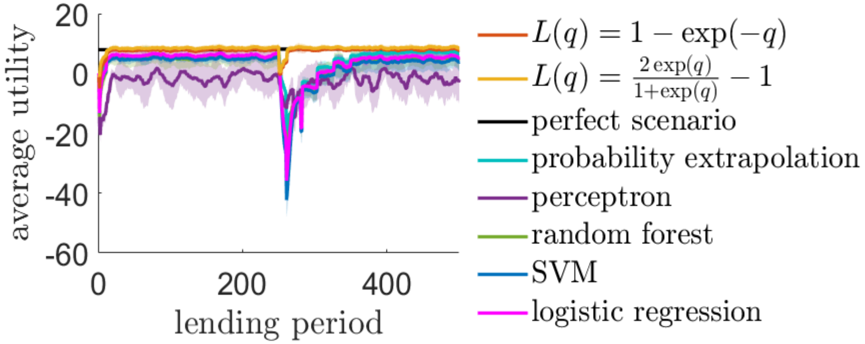
3. **Tradeoff between default rate vs. approval rate**

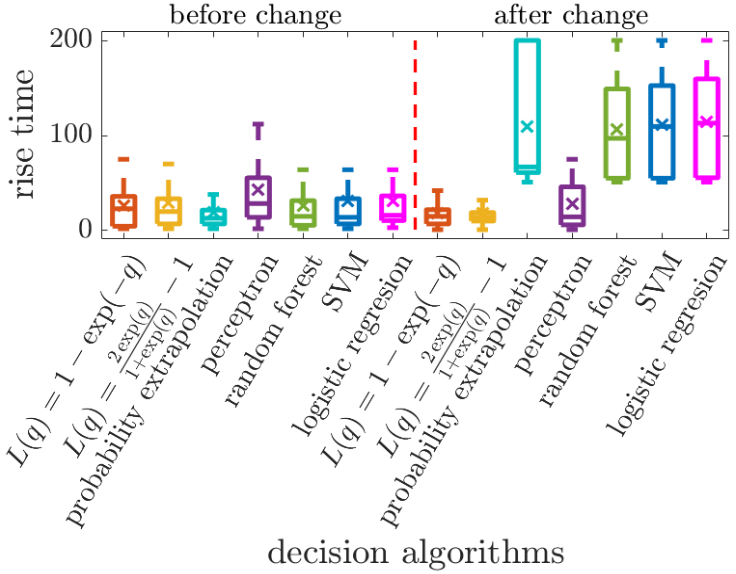


4. **Cheaper computational cost**



5. **Adaptation to changes**





Today’s talk

科学

Neuroscience
Biomolecular control…

工学

Constraints vs robust performance in human

Sensorimotor control

Neurophysiology

Biking,
eye movement, etc.

↑

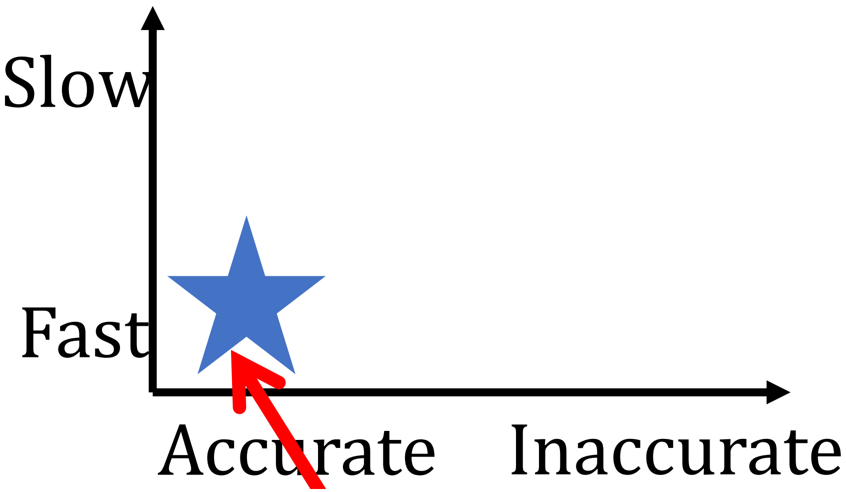
A feedback loop
(e.g. VOR, reflex)

↑

Hardware
(neurons, muscles)

↑

Biological resources



Diversity-enabled sweet spots
through effective architectures