



Carnegie Mellon University
Electrical & Computer Engineering

Adaptive Safe Control for Driving in Uncertain Environments

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Motivation



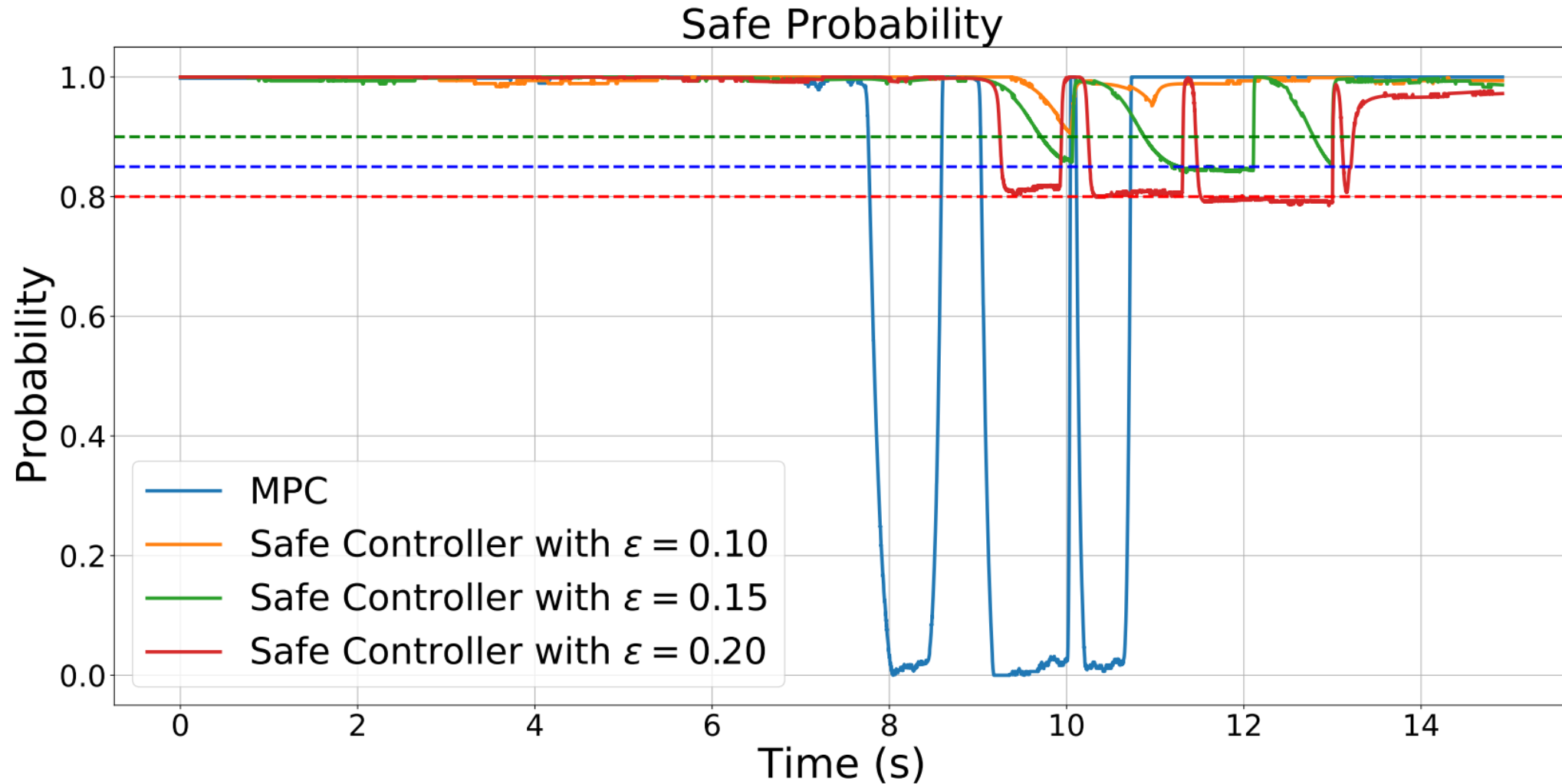
Source: Google Image



- Control algorithm design for autonomous vehicles
- **Robust, adaptive, and computationally efficient**
- Guarantee **long-term safety**
- Work under **large uncertainties** and **changing environments.**

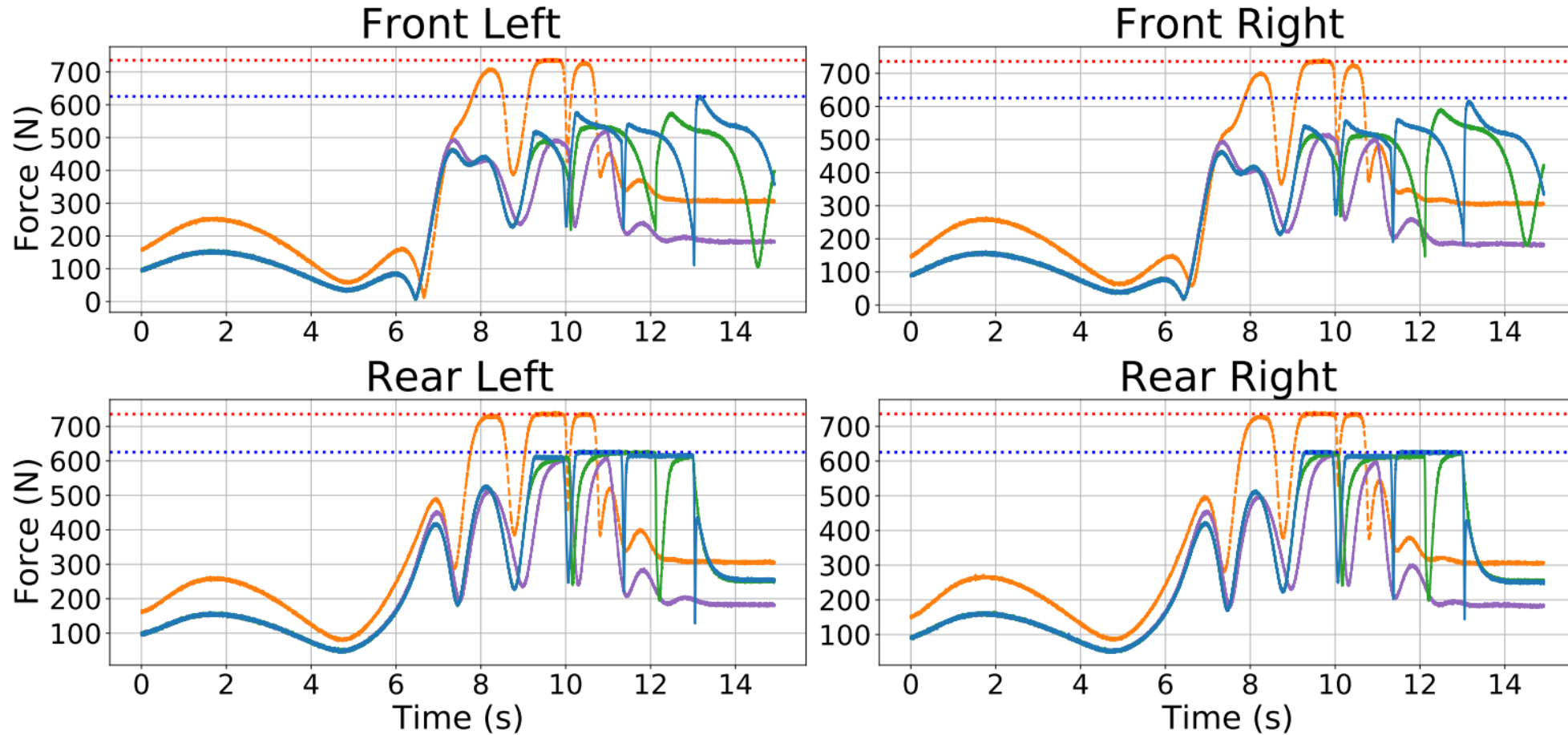
- **Probabilistic safe controls:** over conservatism from attempting robust behaviors to worst-case errors [1-3].
- **Control Barrier/Lyapunov Functions:** difficulty in integrating competing safety v.s. performance objectives [4,5].
- **Reachability based safety:** stringent time-horizon vs computation/reaction-time tradeoffs [6,7].

Advantage 1: Long-term Safety Guarantee



Advantage 1: Long-term Safety Guarantee (Cont'd)

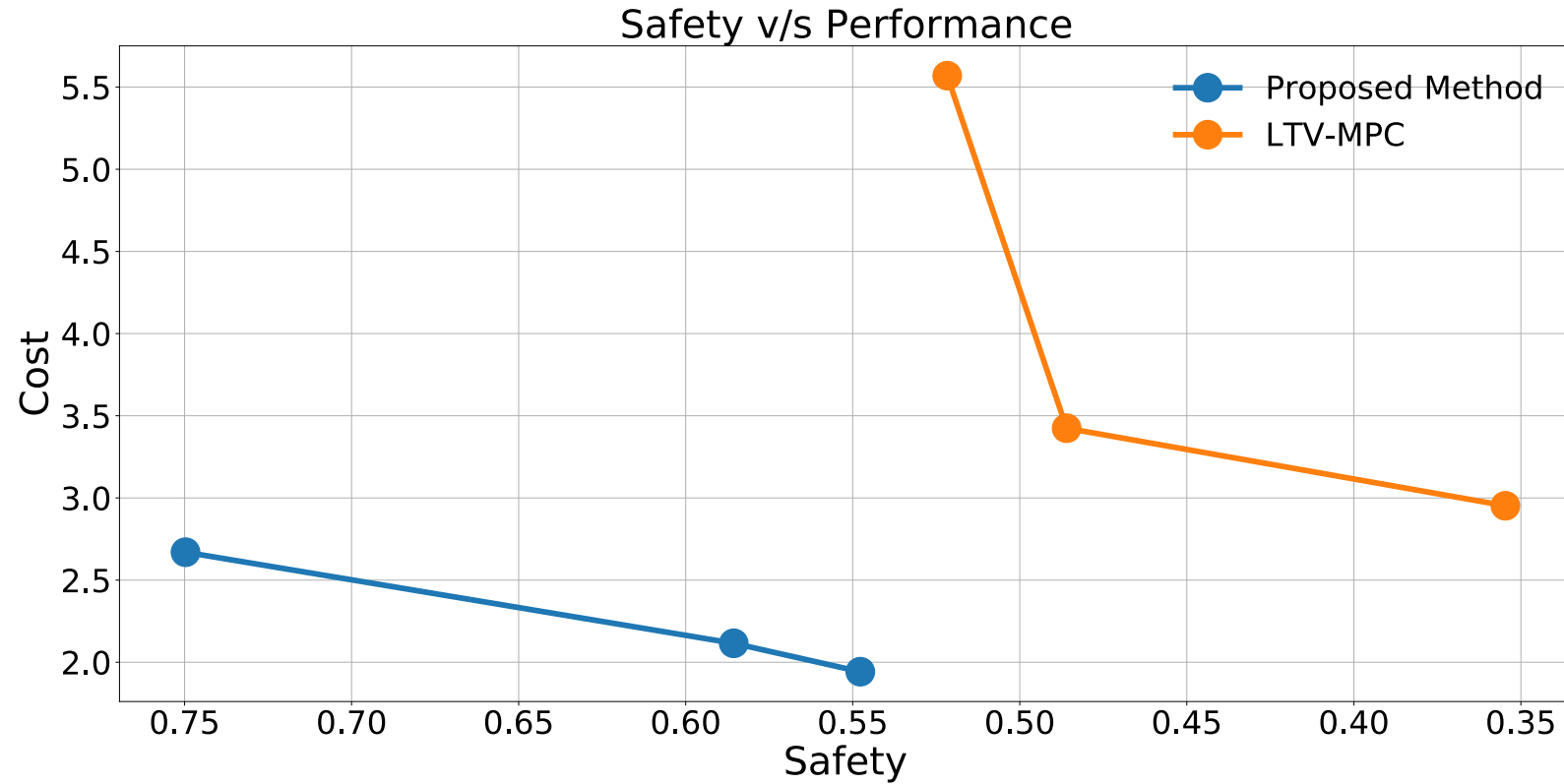
Total Tire Forces



- LTV-MPC
- Proposed Method with $\epsilon = 0.10$
- Proposed Method with $\epsilon = 0.15$
- Proposed Method with $\epsilon = 0.20$
- ⋯ Maximum Tire Grip Force F_{sat}
- ⋯ 85% Maximum Tire Grip Force F_{sat}

Advantage 2: Better Performance Tradeoffs

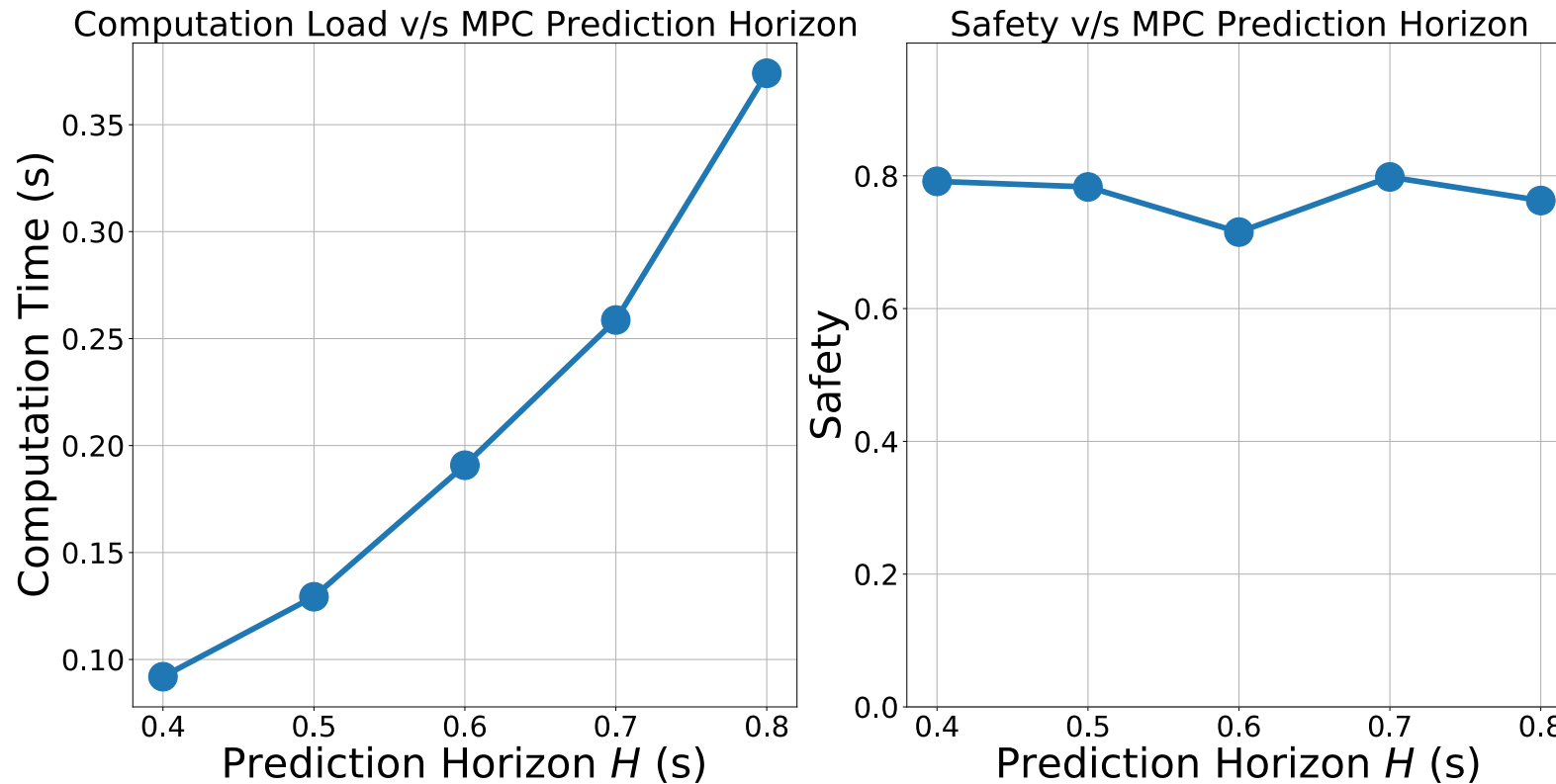
cost:
deviation from
the reference
trajectory



safety: satisfaction of the tire force limits

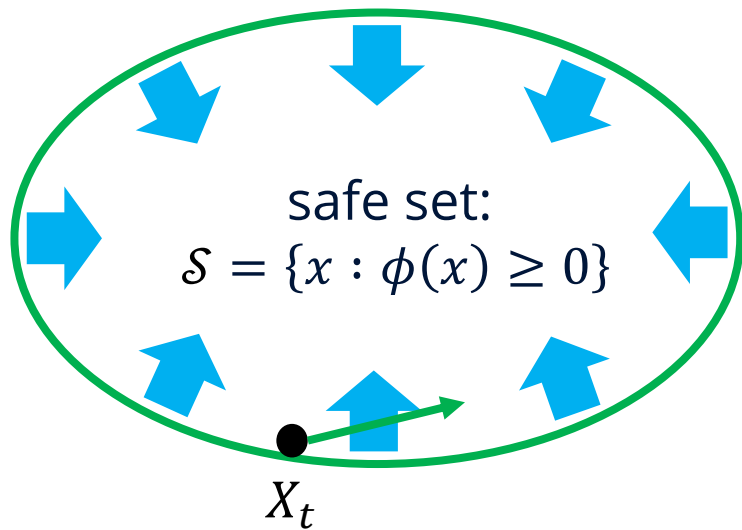
Advantage 3: Less Computation Costs

- Computation of MPC grows in $O(H^3)$
- Safety will not be compromised even with short outlook horizons

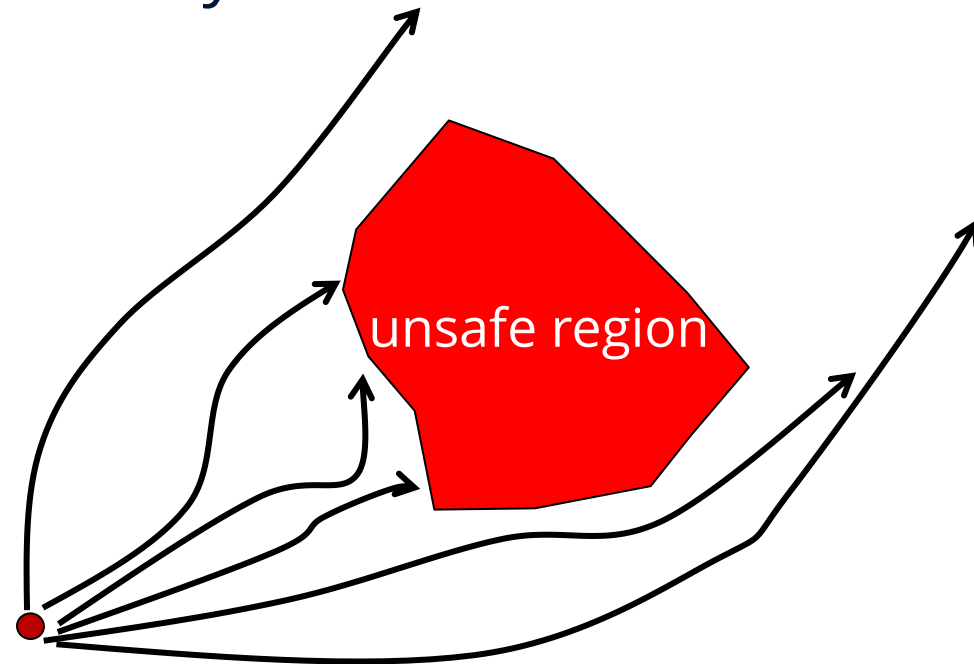


Proposed Method: Intuition

Control barrier functions:

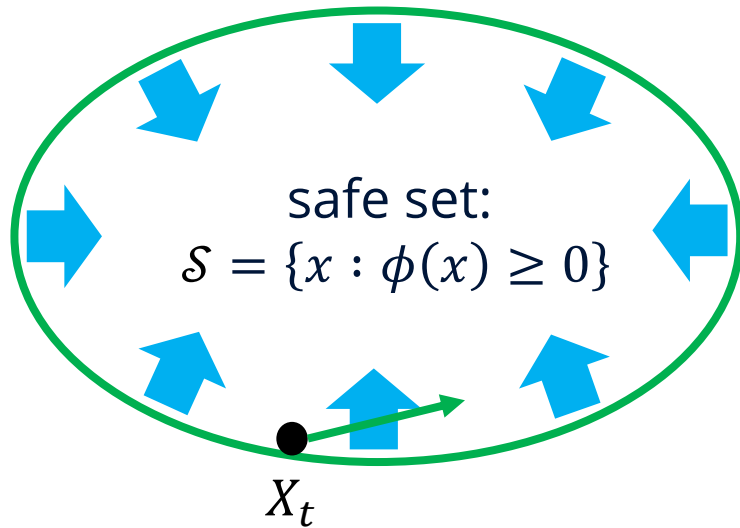


Reachability:

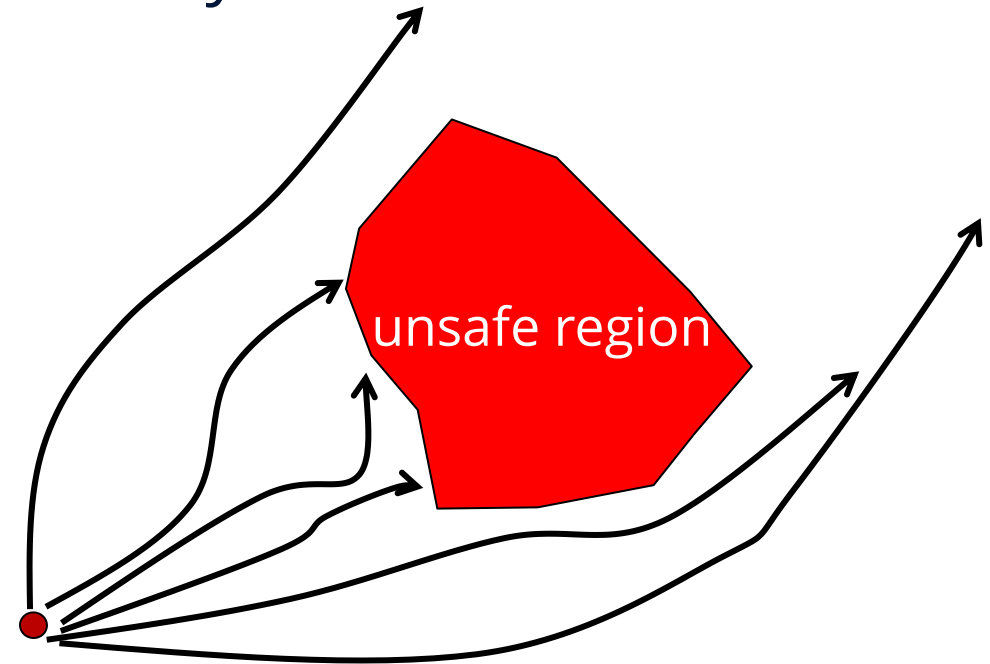


Proposed Method: Intuition

Control barrier functions:



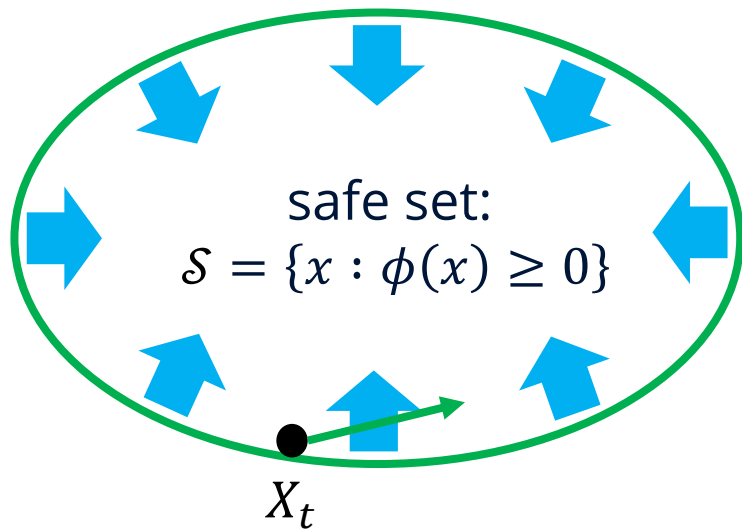
Reachability:



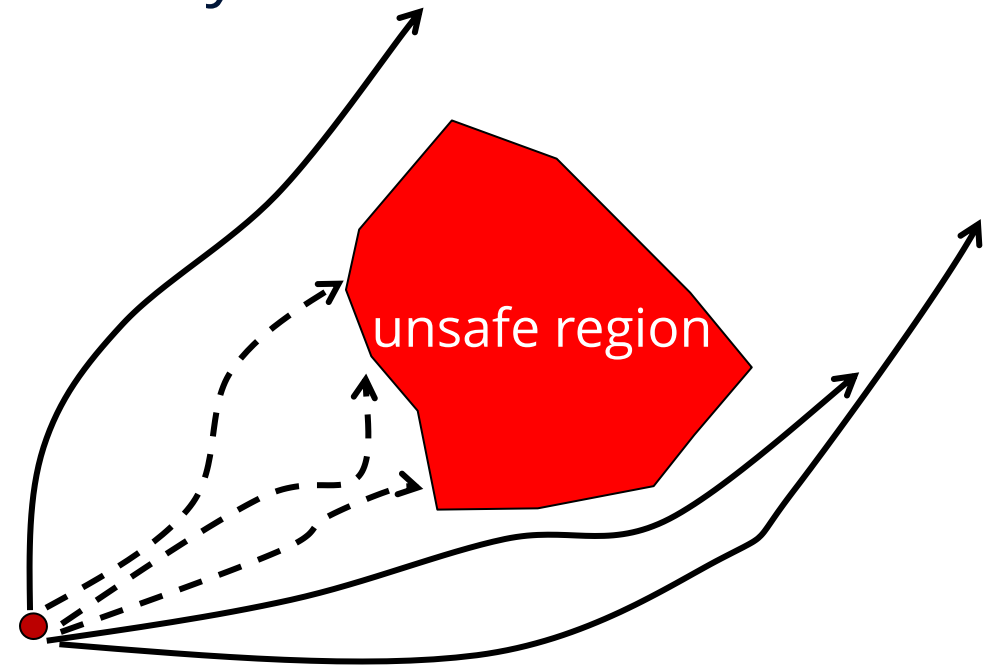
Forward rollout trajectories

Proposed Method: Intuition

Control barrier functions:



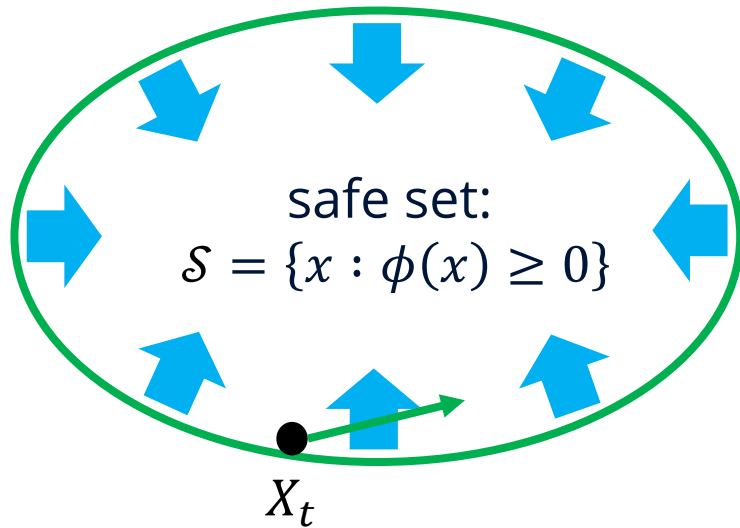
Reachability:



Forward rollout trajectories

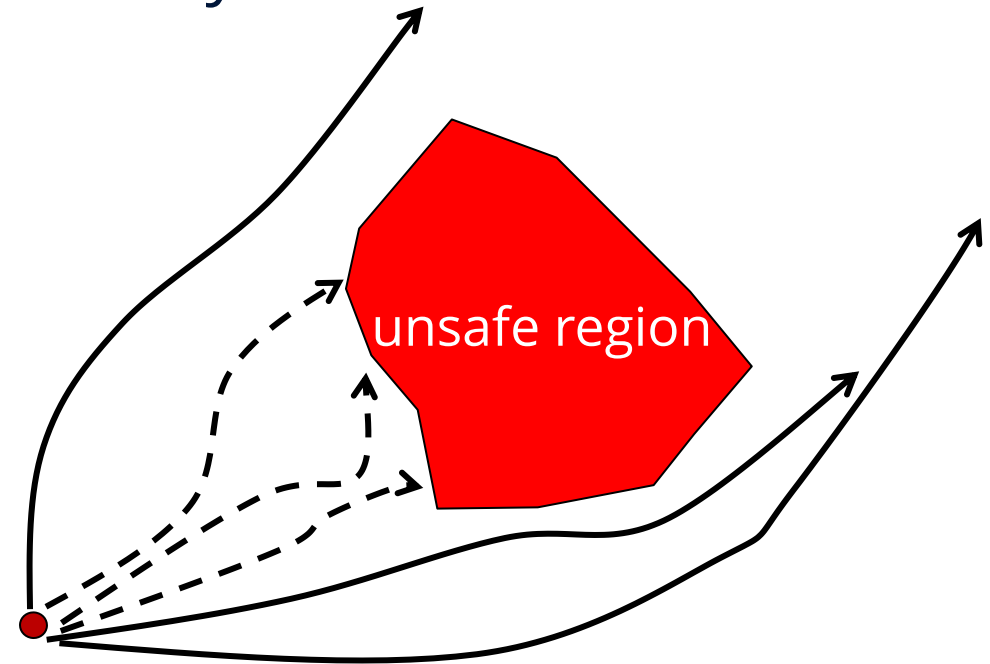
Proposed Method: Intuition

Control barrier functions:



Encoded safety probability $\Psi(X_t)$

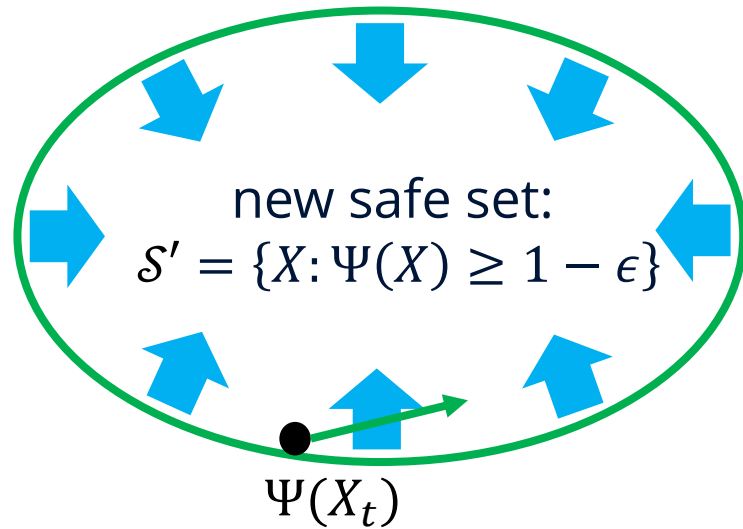
Reachability:



← Forward rollout trajectories

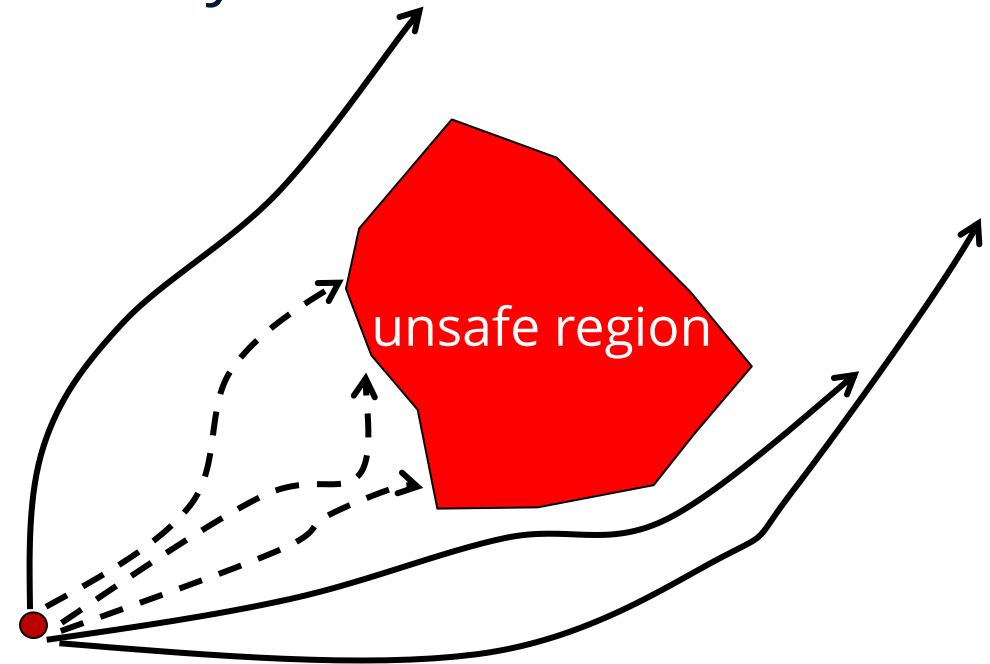
Proposed Method: Intuition

Control barrier functions:



Encoded safety probability $\Psi(X_t)$

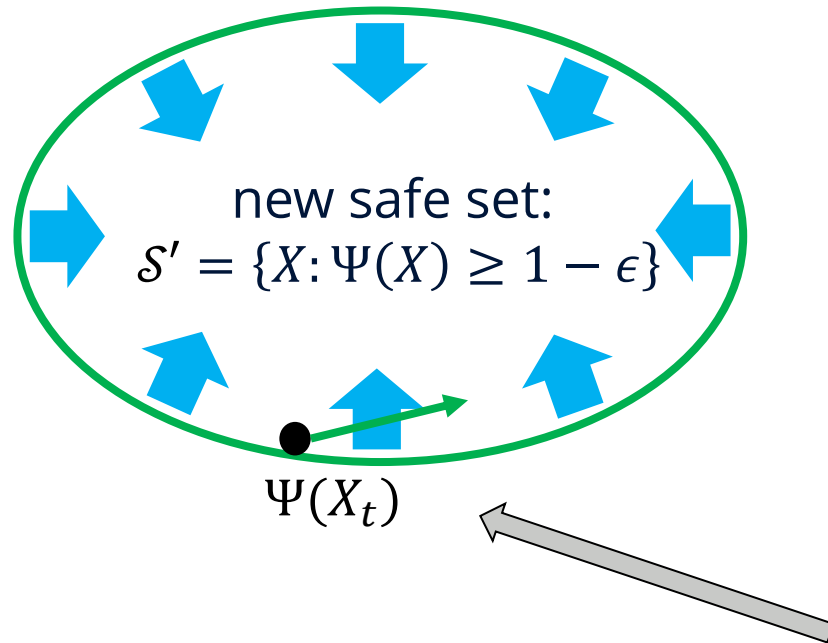
Reachability:



← Forward rollout trajectories

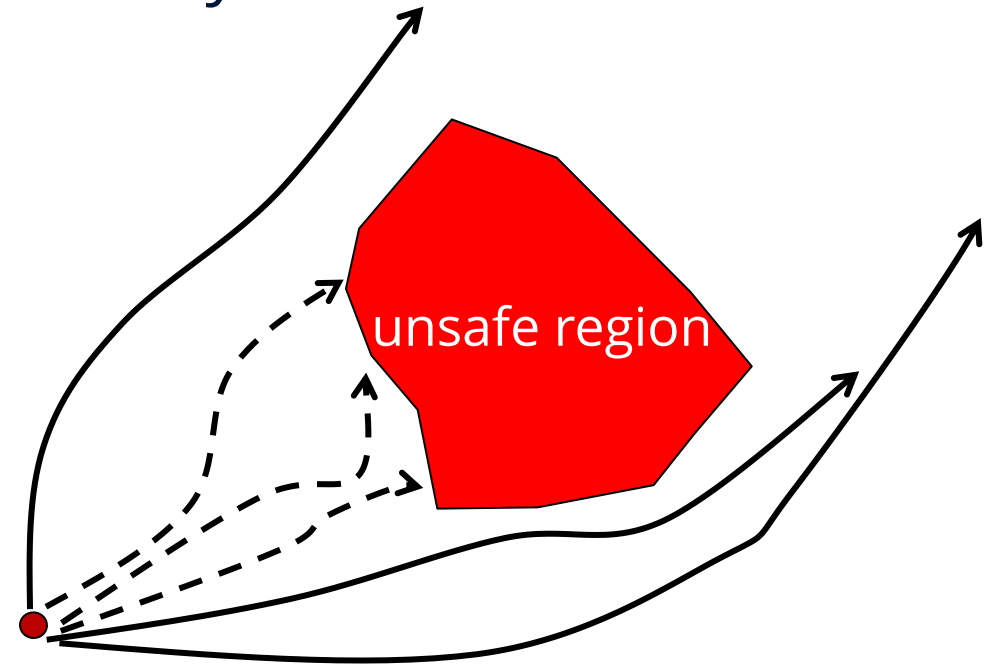
Proposed Method: Intuition

Control barrier functions:



Encoded safety probability $\Psi(X_t)$

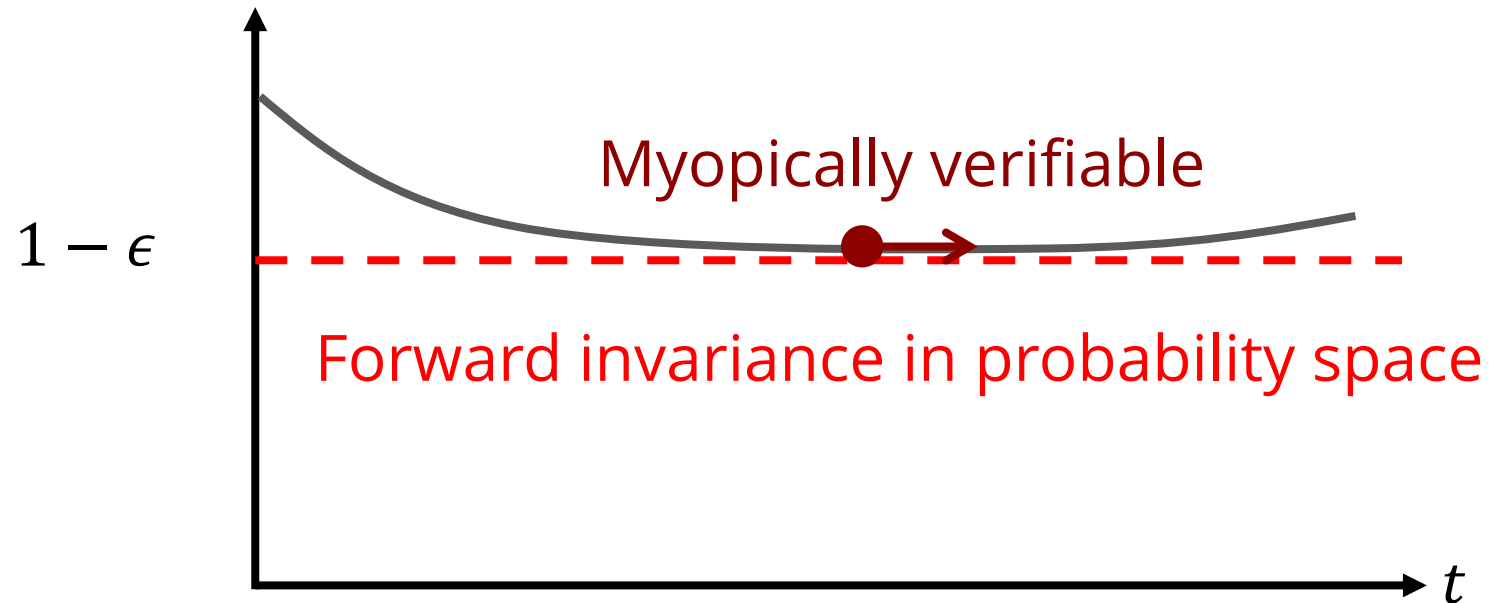
Reachability:



← Forward rollout trajectories

Long-term safe probability

$$\Psi(X_t) = \Pr(X_\tau \in \mathcal{C}, \tau \in [t, t + T] | X_t, \xi_t)$$



Reliability

$$\Psi(X_t) = \Pr(X_\tau \in \mathcal{S}, \tau \in [t, t + T] | X_t, \xi_t)$$

$$A\Psi(X_t) \geq -\gamma(\Psi(X_t) - (1 - \epsilon))$$

time derivative of
safe probability

desired safe
probability

$\gamma: \mathbb{R} \rightarrow \mathbb{R}$ is a concave function, with $\gamma(q) \leq q, \forall q \in \mathbb{R}$.

$$A\Psi(X_t) \geq -\gamma(\Psi(X_t) - (1 - \epsilon))$$

linear with respect to u

$$A\Psi(X_t) = \mathcal{L}_f\Psi(X_t) + (\mathcal{L}_g\Psi(X_t))u + \frac{1}{2}\text{tr}([\sigma(X_t)]^\top \text{Hess}\Psi(X_t)[\sigma(X_t)])$$

constant given system dynamics
 $dX_t = (f(X_t) + g(X_t)U_t)dt + \sigma(X_t)dW$

Efficiency

nominal controller $U_t = N(X_t)$ ensures desired performance without considering safety

The proposed safe controller:

$$U_t = \arg \min_u J(N(X_t), u)$$
$$\text{s. t. } A\Psi(X_t) \geq -\gamma(\Psi(X_t) - (1 - \epsilon))$$

Objective function that penalizes derivation from desired performance, constrained by safety condition.

Modularity

Theme: Safe control strategy for vehicle lateral force control

Features:

- **Provable long-term safety guarantee**
- **Can deal with uncertainties and adapt to changes**
- **Better trade-off between performance, safety, and computation**

Thanks for listening!

Reference

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