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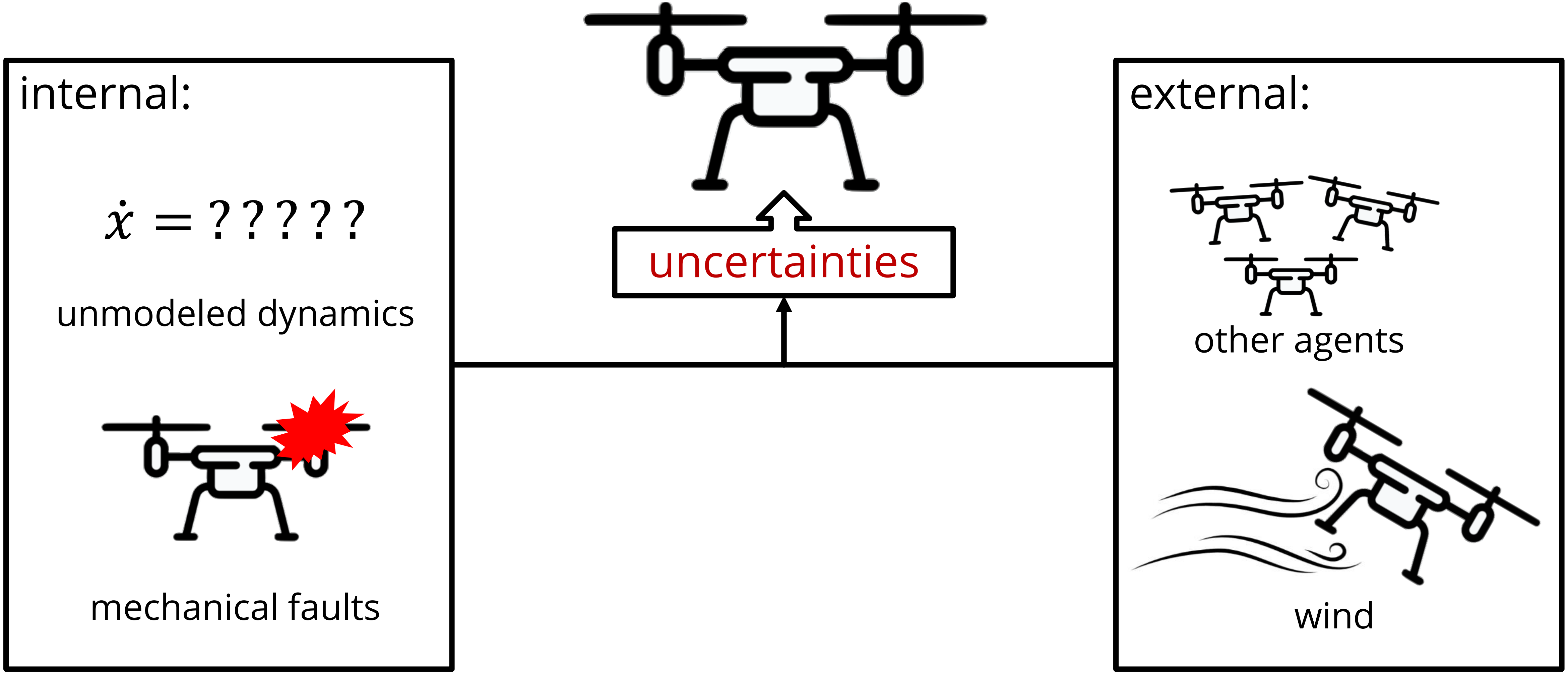
Electrical & Computer Engineering

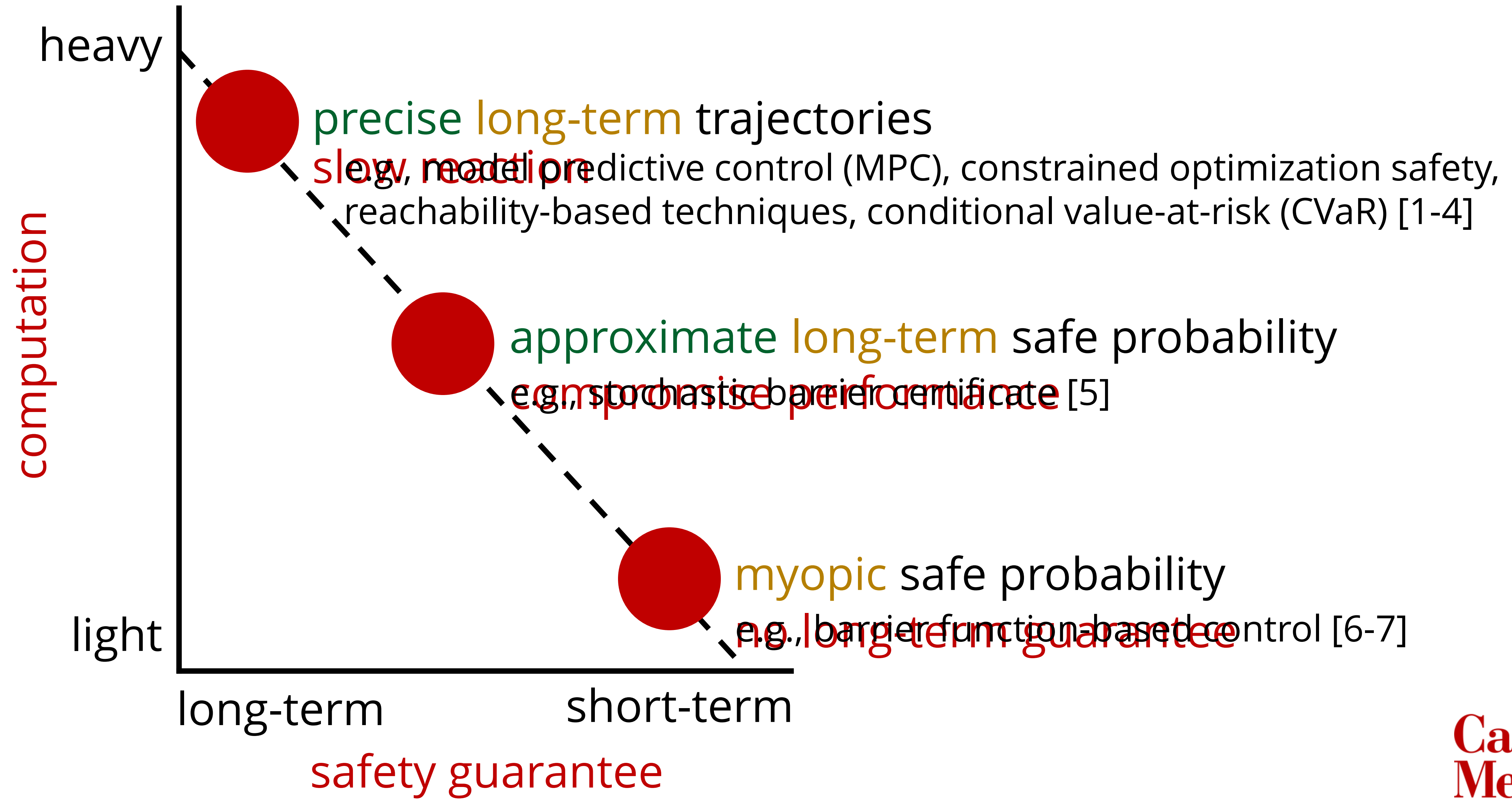
Myopically Verifiable Probabilistic Certificate for Long-term Safety

***Zhuoyuan (Jacob) Wang**, Haoming Jing, Christian Kurniawan, Albert Chern and Yorie Nakahira*

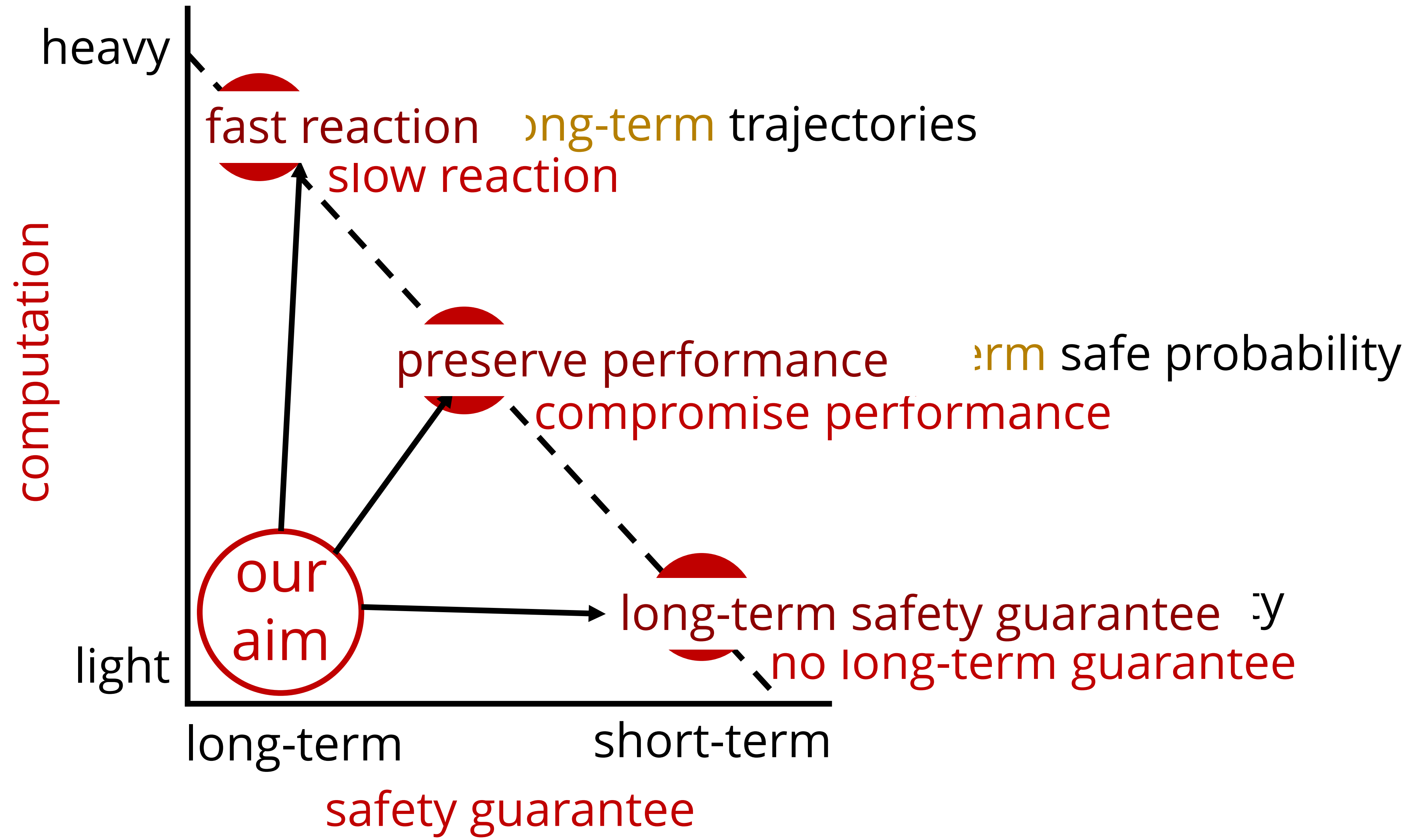
Motivation


internal:
ẋ = ?????
unmodeled dynamics
mechanical faults
uncertainties
external:
other agents
wind

Current Challenges



Aims



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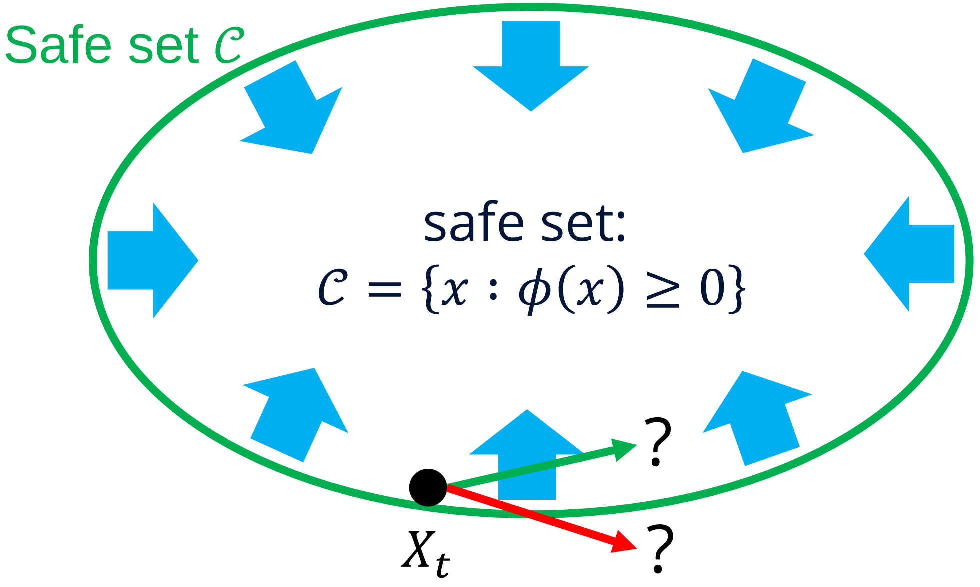
Proposed Method: Intuitions

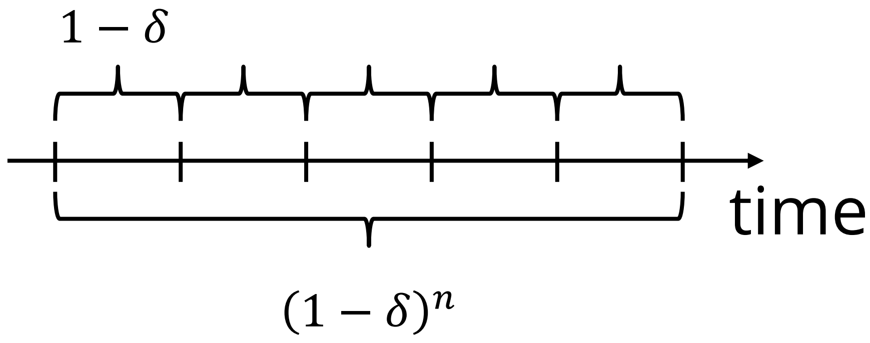
Existing approach:
Control barrier function...



Under stochastic uncertainties

safe at next time => safe at all time ✗

safe with probability $1 - \delta$ at each step



unsafe with high probability in a long term

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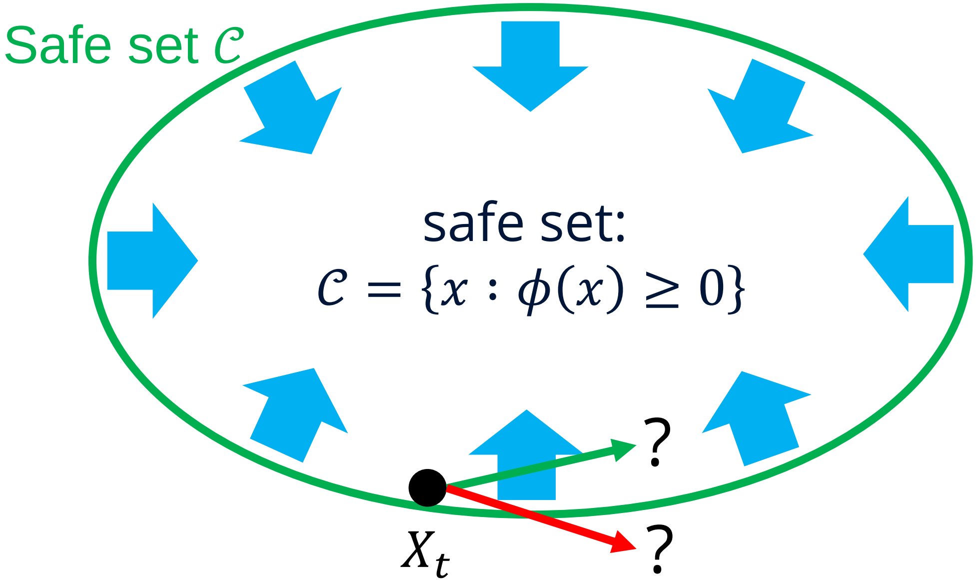
Proposed Method: Intuitions

Existing approach:
Control barrier function…

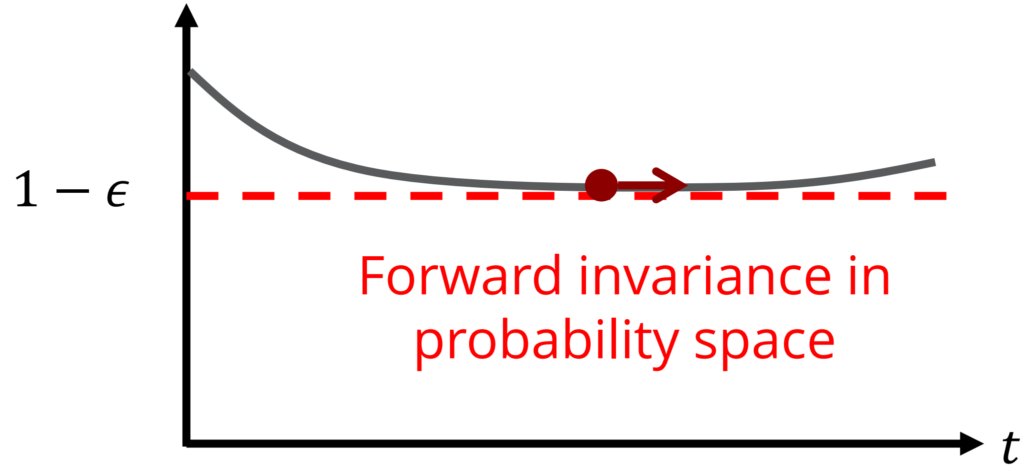


Under stochastic uncertainties

Proposed approach:

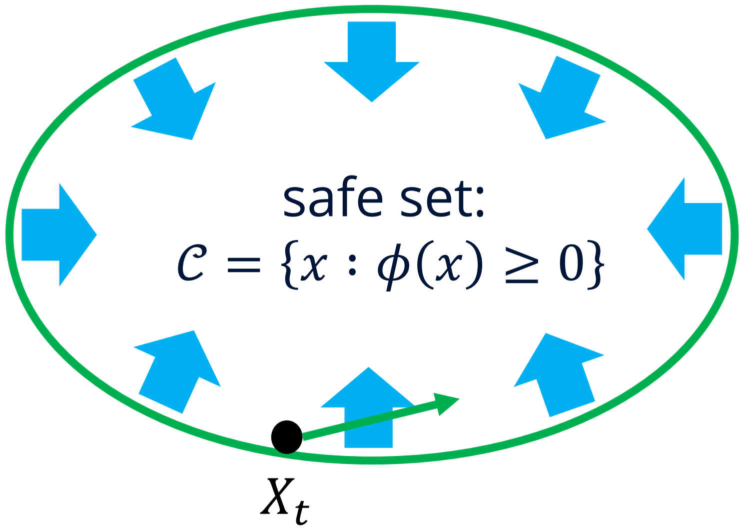
Long-term safety probability

$$\boldsymbol{F}(X_t) = \Pr(X_\tau \in \mathcal{C}, \tau \in [t, t+T] | X_t)$$



Proposed Method: Intuition

Control barrier functions:



Reachability:

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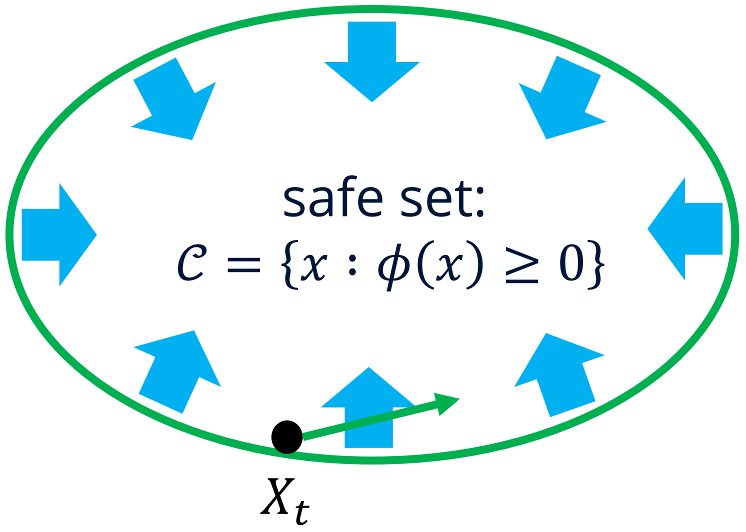
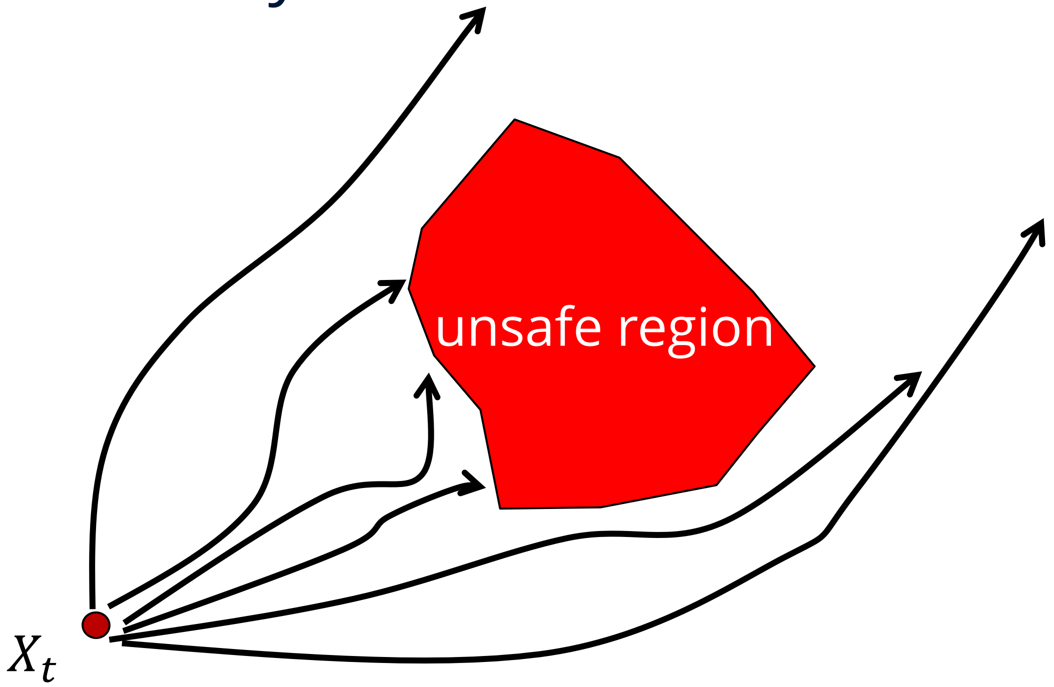
Proposed Method: Intuition

Control barrier functions:



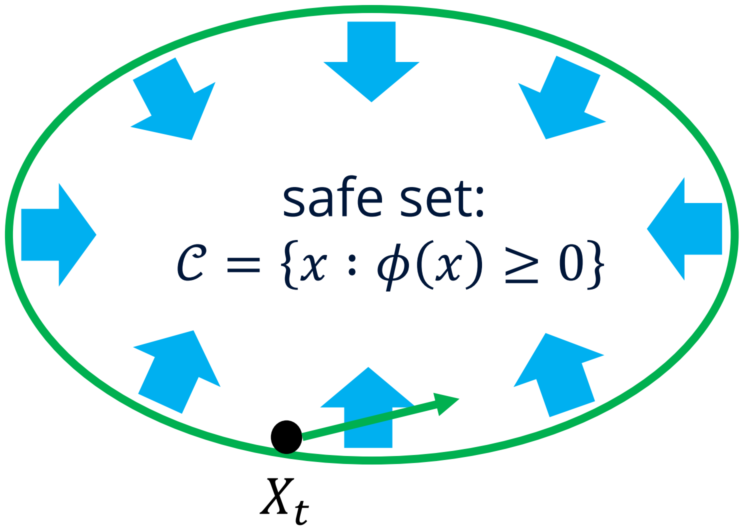
Reachability:



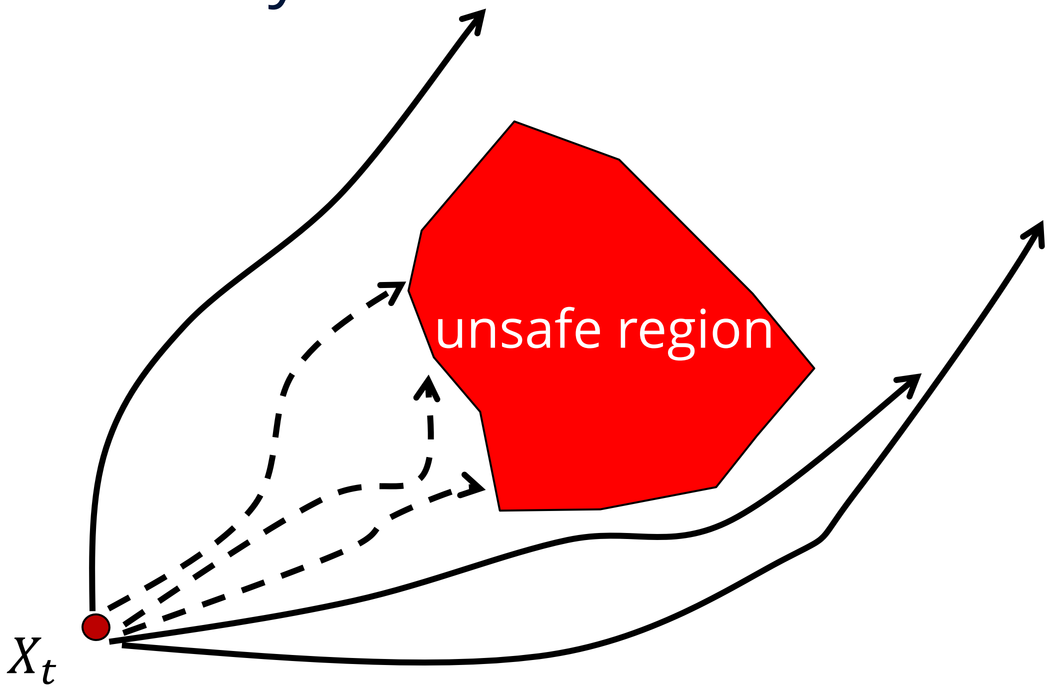
Forward rollout trajectories

Proposed Method: Intuition

Control barrier functions:



Reachability:



Forward rollout trajectories

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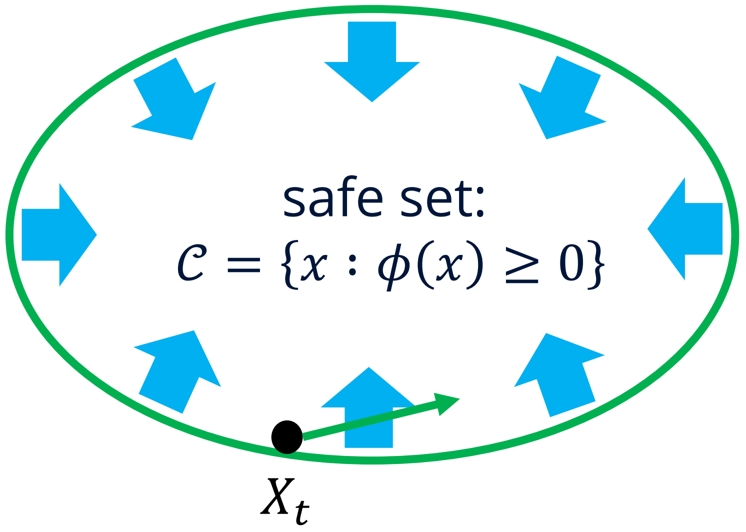
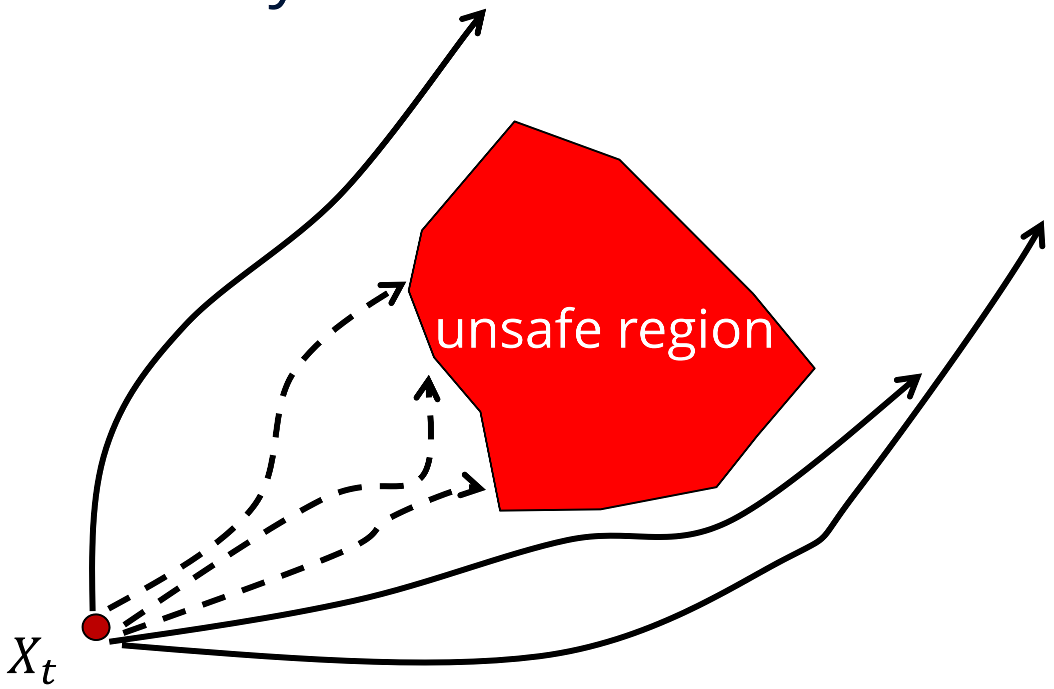


Reachability:



Encoded safety probability $\boldsymbol{F}(X_t)$ ⟵ Forward rollout trajectories

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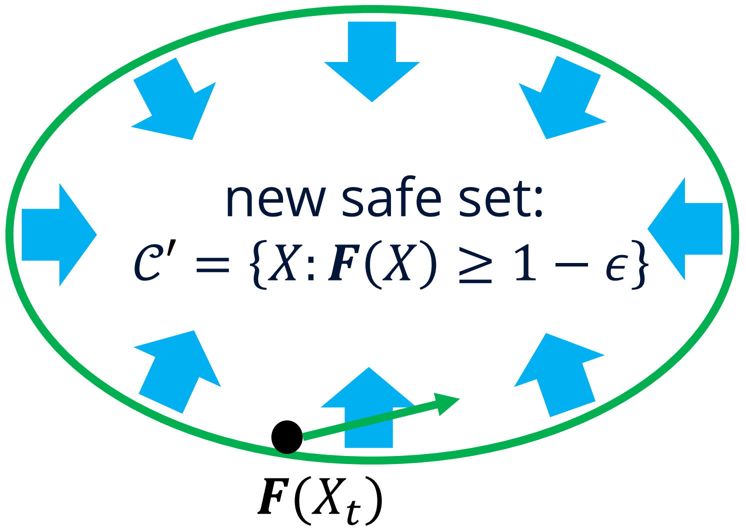
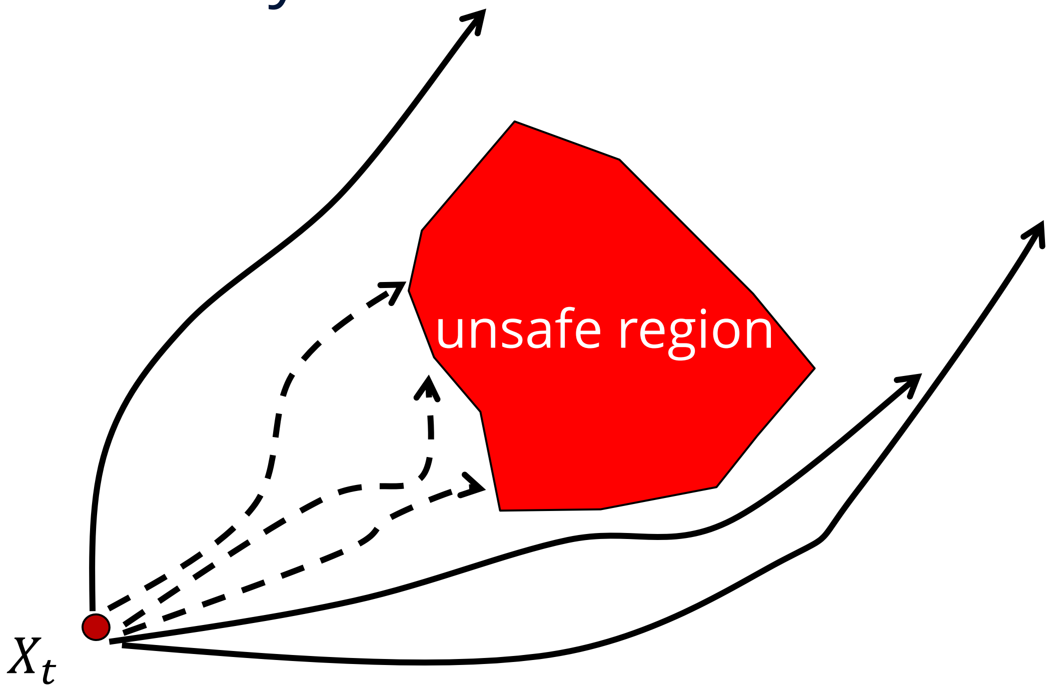
Proposed Method: Intuition

Control barrier functions:



Reachability:



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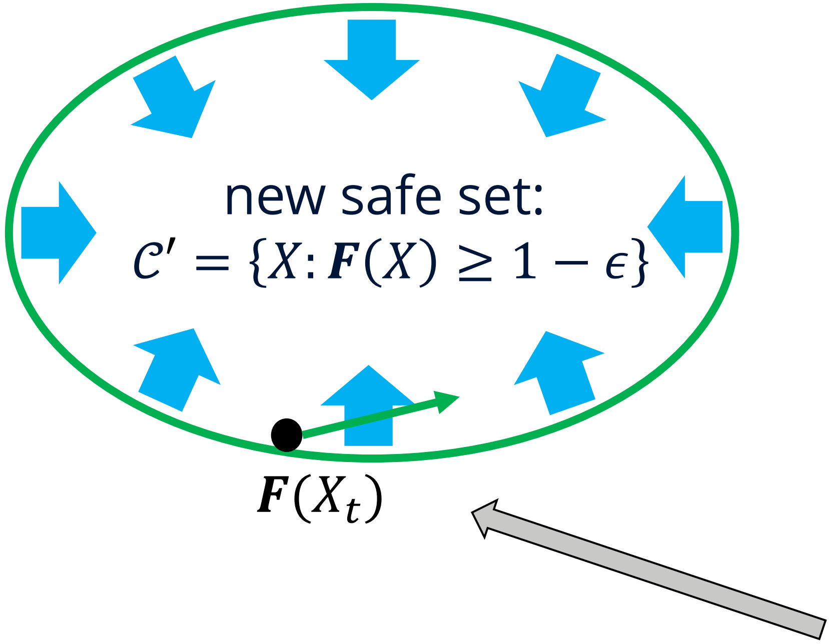
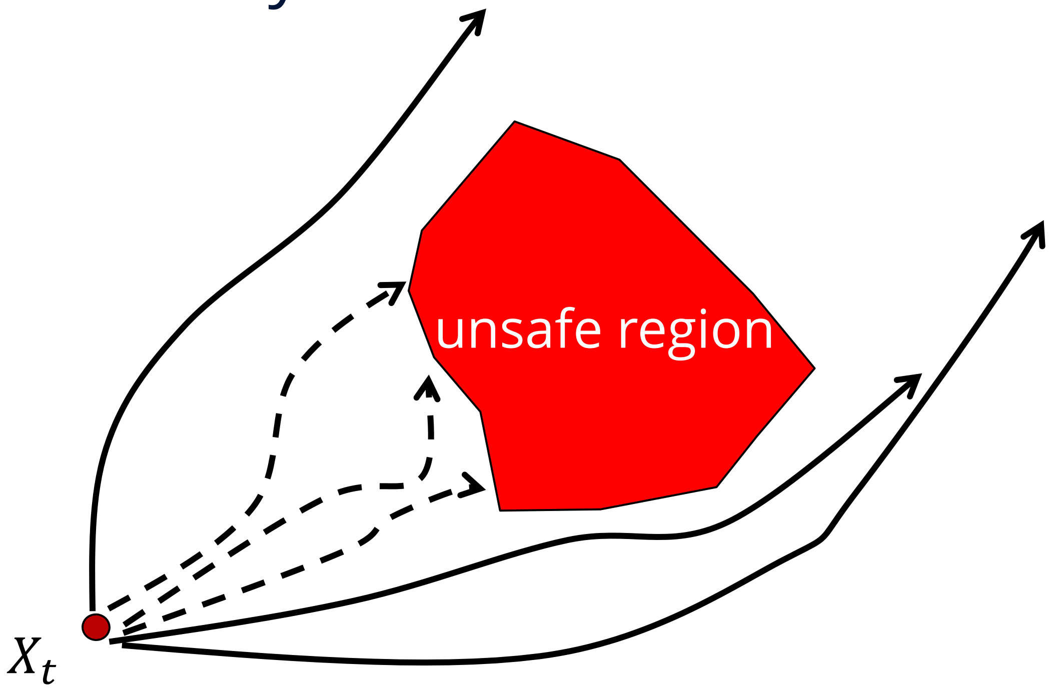


Reachability:

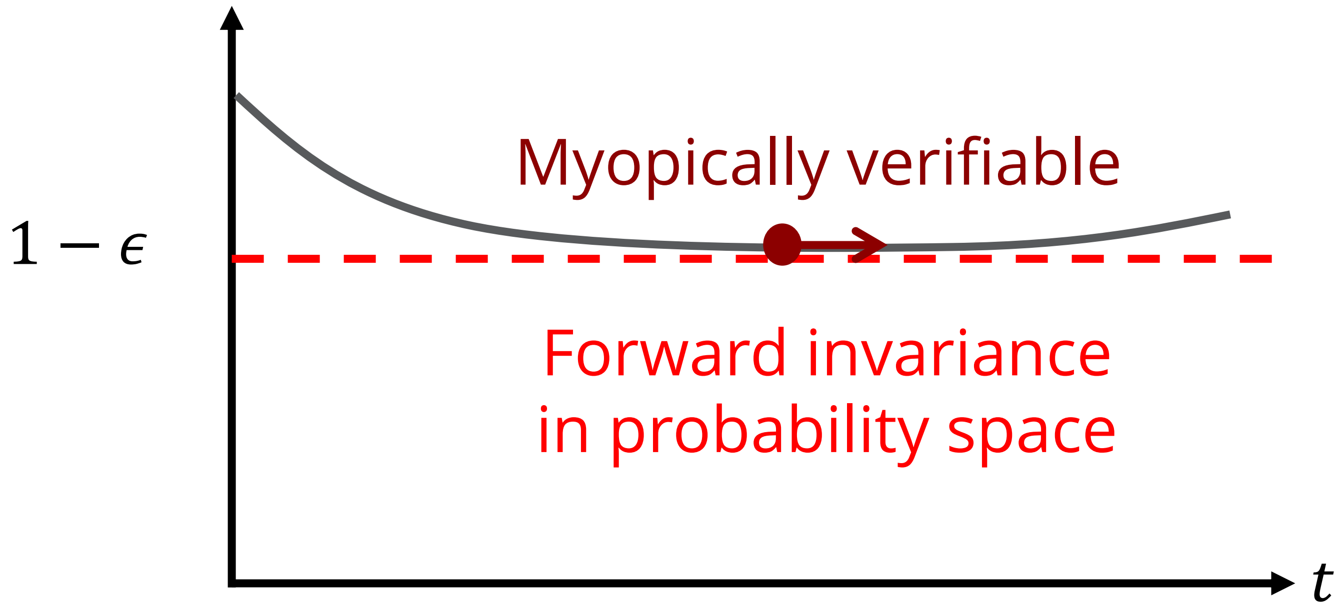


Encoded safety probability $\boldsymbol{F}(X_t)$ ⬅ Forward rollout trajectories

Proposed Method

Long-term safe probability

$$\boldsymbol{F}(X_t) = \Pr(X_\tau \in \mathcal{C}, \tau \in [t, t+T] | X_t)$$



Proposed Safety Condition:

$$A\boldsymbol{F}(X_t) \geq -\alpha(\boldsymbol{F}(X_t) - (1 - \epsilon))$$

$A\boldsymbol{F}(X_t)$: time derivative of safety probability

$(1 - \epsilon)$: desired safety probability

A: infinitesimal generator

$\alpha: \mathbb{R} \to \mathbb{R}$ monotonically increasing, concave, $\alpha(0) \leq 0$.

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Proposed Safety Condition

$$\mathcal{A}\boldsymbol{F}(X_t) \geq -\alpha(\boldsymbol{F}(X_t) - (1-\epsilon))$$

linear with respect to u

$$\mathcal{A}\boldsymbol{F}(X_t) = \mathcal{L}_f\boldsymbol{F}(X_t) + \left(\mathcal{L}_g\boldsymbol{F}(X_t)\right)u + \frac{1}{2}\mathrm{tr}([\sigma(X_t)]^\intercal \mathrm{Hess}\boldsymbol{F}(X_t)[\sigma(X_t)])$$

constant given system dynamics

$$dX_t = (f(X_t) + g(X_t)U_t)dt + \sigma(X_t)dW$$

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Theoretical Guarantees

Theorem: Given

$$F(X_0) > 1 - \epsilon,$$

if we choose the control action to satisfy

$$AF(X_t) \geq -\alpha(F(X_t) - (1-\epsilon)) \text{ for } t > 0,$$

then we have

$$\Pr(X_\tau \in \mathcal{C}, \tau \in [t, t+T]) \geq 1 - \epsilon \text{ for } \forall t > 0$$

$\alpha: \mathbb{R} \to \mathbb{R}$ is a monotonically increasing concave function that satisfies $\alpha(0) \leq 0$.

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Wang et al., (2021).

Simulation

system dynamic: $dx_t = (2x_t + 2.5u_t)\,dt + 2dw_t$

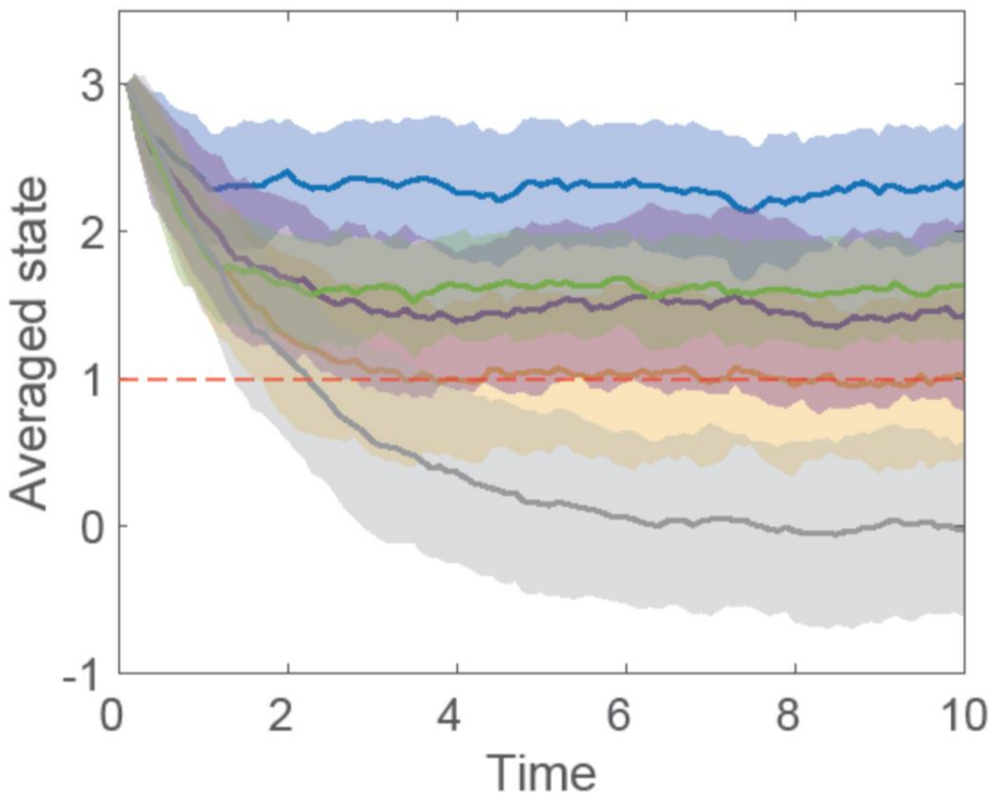
initial state: $x_0 = 3$

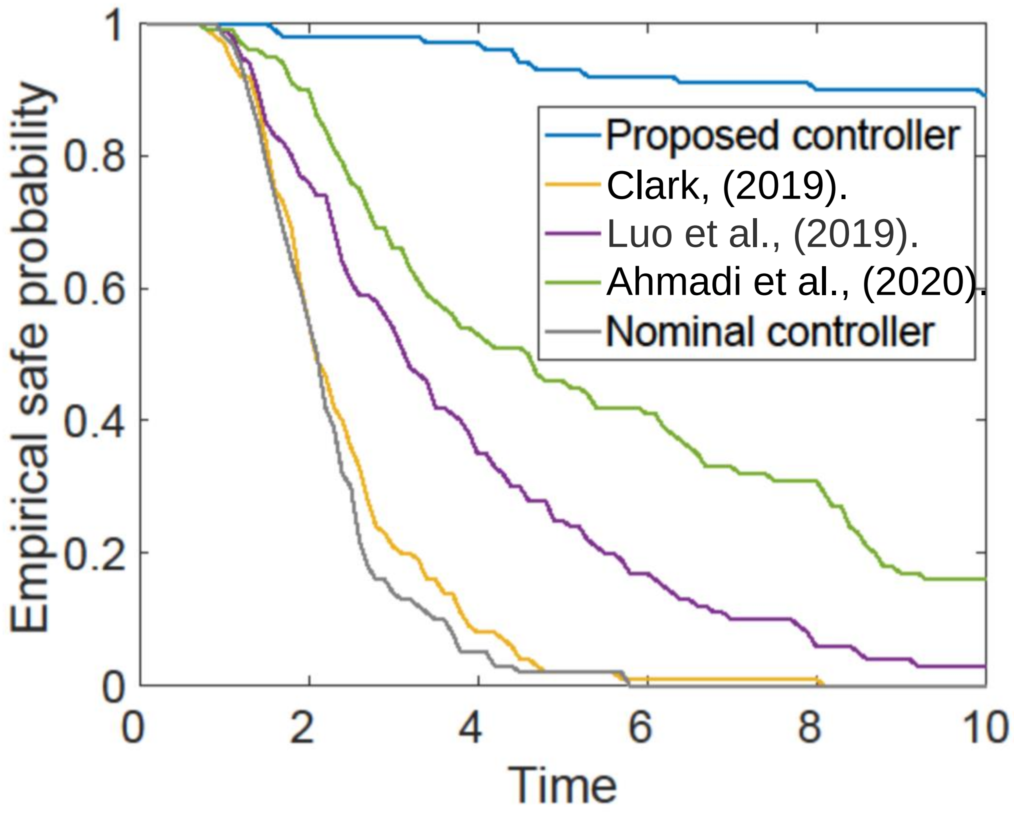
safe set: $\mathcal{C} = \{x \in \mathbb{R} : x - 1 > 0\}$

nominal controller: $N(x_t) = 2.5x_t$

desired safety probability: $1 - \epsilon = 0.9$

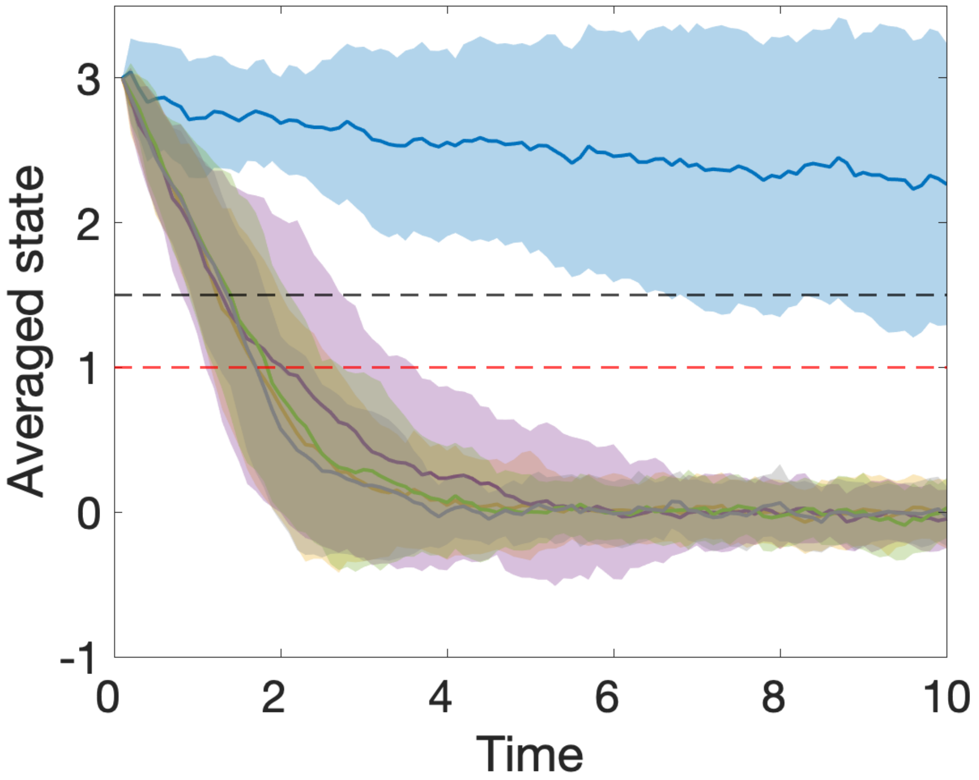
Simulation

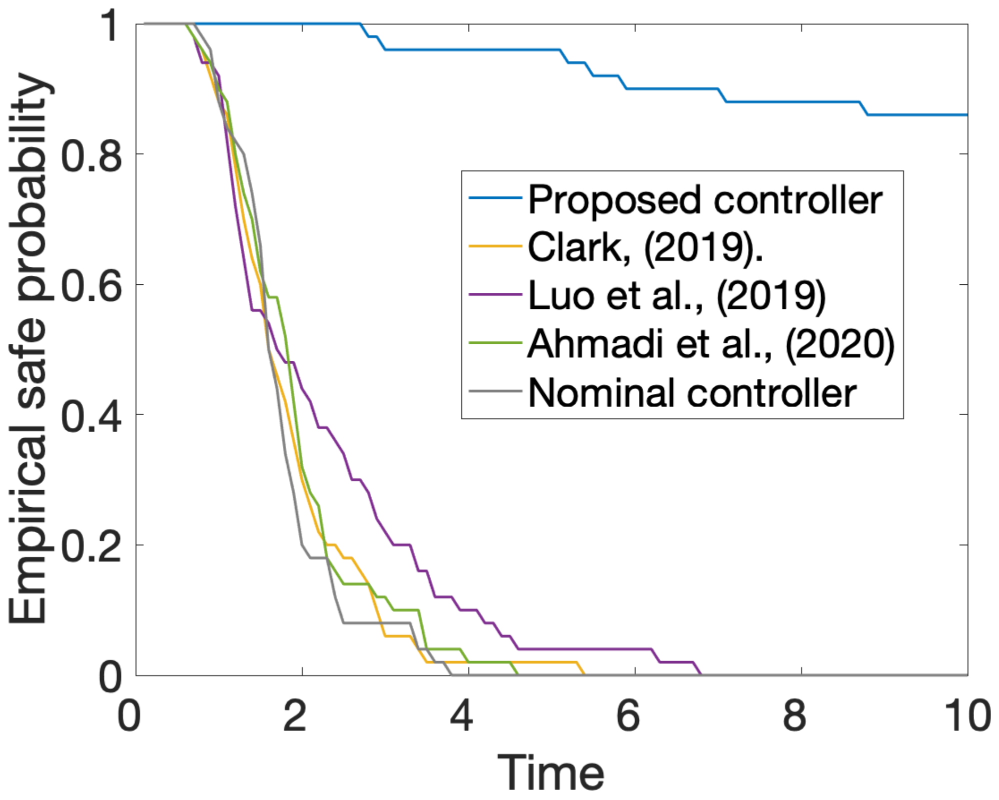

Averaged state
3
2
1
0
-1
0
2
4
6
8
10
Time


Empirical safe probability
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
Time
Proposed controller
Clark, (2019).
Luo et al., (2019).
Ahmadi et al., (2020).
Nominal controller

Simulation – Nonlinear trap

System becomes uncontrollable once reach state $x = 1.5$





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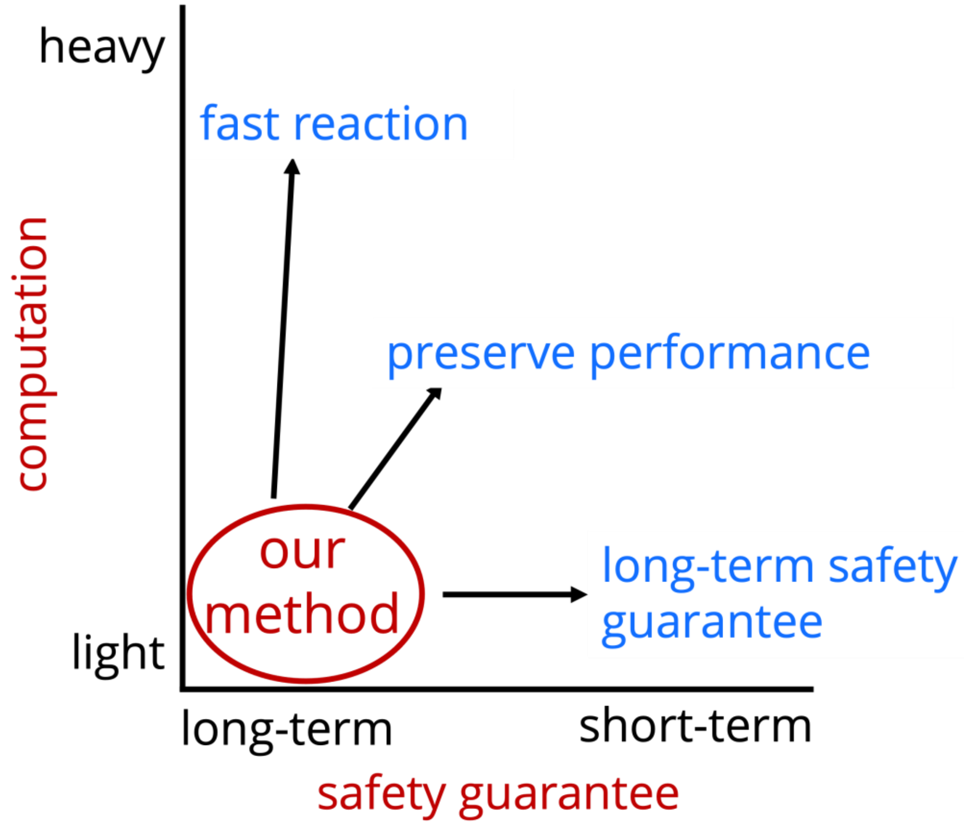
Conclusion



- **Provable long-term safety guarantee**
- **Fast reaction with reduced computation**
- **Controllable safety and performance trade-off**
- **Easy implementation with plug-in usage**

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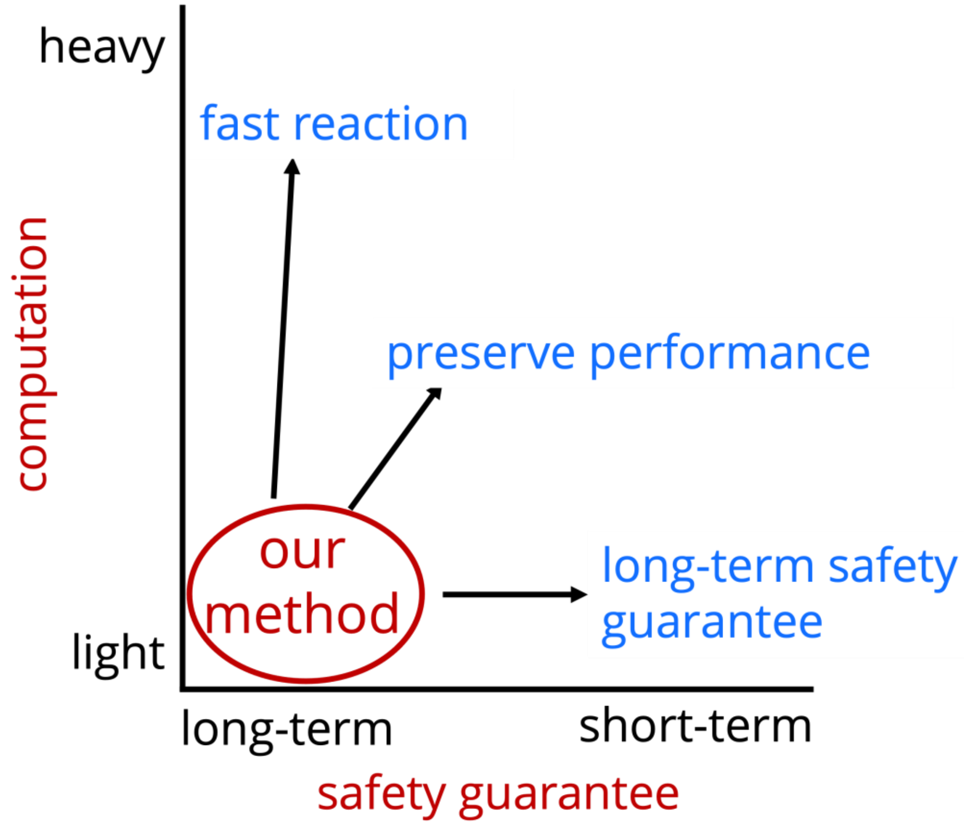
Future Directions



- **Faster and more accurate estimation of safety probability**
- **Compact representation of the safety probability mapping (e.g. DNN)**
- **…**

Thanks for listening!

Scan for full manuscript on arXiv

Application to Autonomous Vehicles

S. Gangadhar, Z. Wang, H. Jing, and Y. Nakahira, "Adaptive Safe Control for Driving in Uncertain Environments," *the 33rd IEEE Intelligent Vehicles Symposium (IV),* 2022.

Generalization to Distributed Systems

H. Jing, and Y. Nakahira, "Probabilistic safety certificate for multi-agent systems," *submitted to 2022 Control and Decision Conference (CDC)*, 2022.

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Reference

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[2] Ahmadi, Mohamadreza, Xiaobin Xiong, and Aaron D. Ames. "Risk-averse control via CVaR barrier functions: Application to bipedal robot locomotion." *IEEE Control Systems Letters* 6 (2021): 878-883.

[3] Bansal, Somil, et al. "Hamilton-jacobi reachability: A brief overview and recent advances." *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*. IEEE, 2017.

[4] Oishi, Meeko, et al. "Addressing multiobjective control: Safety and performance through constrained optimization." *International Workshop on Hybrid Systems: Computation and Control*. Springer, Berlin, Heidelberg, 2001.

[5] Prajna, Stephen, Ali Jadbabaie, and George J. Pappas. "Stochastic safety verification using barrier certificates." *2004 43rd IEEE conference on decision and control (CDC)(IEEE Cat. No. 04CH37601)*. Vol. 1. IEEE, 2004.

[6] Clark, Andrew. "Control barrier functions for complete and incomplete information stochastic systems." *2019 American Control Conference (ACC)*. IEEE, 2019.

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[7] Luo, Wenhao, Wen Sun, and Ashish Kapoor. "Multi-robot collision avoidance under uncertainty with probabilistic safety barrier certificates." *Advances in Neural Information Processing Systems* 33 (2020): 372-383.

[8] Wang, Zhuoyuan, et al. "Myopically Verifiable Probabilistic Certificate for Long-term Safety." *arXiv preprint arXiv:2110.13380* (2021).

[9] Chern, Albert, et al. "Safe Control in the Presence of Stochastic Uncertainties." *arXiv preprint arXiv:2104.01259* (2021).

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