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Control of Automated Vehicles in Vehicle-Pedestrian Environment

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16. Abstract Automated Vehicles (AV's) can intermingle with pedestrians and cyclists when they are driving slowly in so-called "shared spaces". In our previous work, we studied AVs' energy consumption and safety when occluded pedestrians appear suddenly in front of the AV. We will continue investigating and developing our "value of information" based approach to evaluate additional sensors in the infrastructure. One can consider regular intersections and focus on specific configurations. We have initiated a study on "indecisive pedestrians". These are pedestrians who may stop or turn back while crossing the street, depending on their assessment of the approaching vehicle. We assume that the vehicle will also make a decision on stopping, continuing and/or dodging the pedestrian.			
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Chapter 1

Introduction

Vehicle-pedestrian interactions pose a complex safety issue. Ideally, vehicles should react safely to any pedestrian behavior, but pedestrian movement itself can be unpredictable and difficult to model. This challenge is underscored by a 2019 National Highway Traffic Safety Administration (NHTSA) report [1] which highlights an increase in pedestrian fatalities from 2008 to 2017, even amidst the development and deployment of advanced driver-assistance systems (ADAS). As autonomous driving systems progress, pedestrian safety becomes an even more critical concern.

These systems must be adept at handling both common situations, like pedestrians crossing at crosswalks, and less frequent, riskier scenarios, such as a pedestrian emerging unexpectedly from between parked cars. To navigate these interactions effectively, accurate models of pedestrian behavior are crucial.

One prevalent approach within research is the Social Force Model, which helps predict pedestrian movement. This model is particularly valuable because data-driven methods often struggle with rare or risky behaviors due to a lack of training data on such edge cases.

Beyond solely modeling behavior, the vehicle also needs to predict pedestrian intentions. Key questions include: Does the pedestrian intend to cross the road? If so, in which direction? Similar to the adoption of model-predictive control for non-linear control tasks, predicting pedestrian intention can significantly enhance the safety of vehicle-pedestrian interactions. This allows the vehicle to react proactively to pedestrian movements, leading to safer outcomes.

Pedestrians crossing unsignalized intersections, mid-road sections, or jaywalking rely on perceived gaps in traffic, called "time gaps." This refers to the estimated time it takes a vehicle to reach the pedestrian's intended crossing point. Current research prioritizes collecting data on pedestrian behavior in various crossing scenarios. This data is then used to analyze two key aspects:

Time Gap Assessment: How pedestrians judge the available time gap.

Gap Acceptance: Whether pedestrians are willing to cross within that gap.

These analyses are conducted by fitting the data to statistical models, either traditional or machine learning-based. The model's performance in predicting pedestrian behavior is then evaluated. Real-world testing of these scenarios can be dangerous. Therefore, researchers leverage simulation environments to create controlled scenarios for rigorous testing. This approach has evolved from generic simulations to more targeted scenarios that pinpoint the limitations of control systems designed to enhance pedestrian safety. Our research has also utilized scenario generation methods to thoroughly evaluate our

own control system design.

This Report describes how to design Control of Automated Vehicles to minimize pedestrian casualties and accidents. Rest of the report is organized as follows. Chapter two presents a research problem that describes how to assess risk while ego vehicle is making control action to avoid collisions with pedestrians. chapter three explains if encountering indecisive pedestrian how control of ego vehicle can be designed to avoid collision.

Chapter 2

FSM-based Control of Automated Vehicles using Sufficient Conditions and Decision Matrices

This chapter is derived from the work yet to be published.

2.1 Introduction

2.1.1 Simulation Case Study

The case study that is considered to show the general nature and effectiveness of the vehicle and pedestrian guidance system is shown in Fig. 2.1. p and v are pedestrian and vehicle respectively. d_{gap} is the distance between vehicle and pedestrian, d_{cross} is the distance between the pedestrian starting and ending point, d_{back} is the distance between pedestrian and starting point s and d_{remain} is the distance between pedestrian and ending point e . The problem mentioned in the Fig. 2.1, pedestrians can display a range of behaviors such as surprise stop, moving back, and moving forward. The vehicle in Fig. 2.1 can also accelerate, decelerate, dodge and move with constant speed. The main challenge in the problem scenario is to estimate the accurate time taken by a vehicle and a pedestrian to travel a certain distance (d_{cross} and d_{gap}) while their speeds can be variable. The goal is to devise pedestrian and vehicle guidance systems that can assess the possibility of collision and guide pedestrian and vehicle appropriately to avoid a collision. Furthermore, the guidance system can also identify situations when collision avoidance is impossible and will make decisions to not enter in those situations. The simplest case is considered where a pedestrian is moving in the lateral direction and the vehicle is moving in the longitudinal direction. This is a decentralized control problem and vehicles and pedestrians are autonomous entities and can make decisions independently. This case is used to show that a general solution can be used to assess the time gap between pedestrians and vehicles. This time gap can then used in pedestrian and vehicle guidance systems to avoid collisions. Decision Matrices (DMs) are presented to assess appropriate decision-making and when systems would be unable to avoid a collision. These DMs are then used in designing hierarchical controller.

Some assumptions have been considered. when a Vehicle Control System (VCS) is designed it is assumed that pedestrian behavior (forward. backward. wait, speed up)

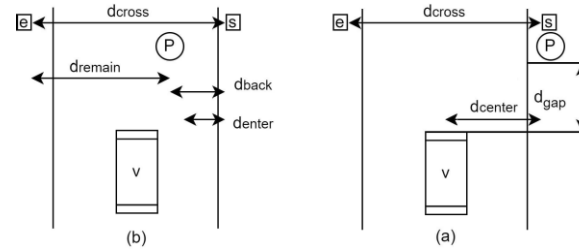


Figure 2.1: Problem Scenario.

is known. The position and speed of vehicles and pedestrians are known and contain a limited amount of error. Speed and position information assumptions are realistic as the latest communication systems such as 4G/5G networks make low-speed communication effective and reliable. Furthermore, it is assumed that vehicle capability (constant speed, brake, hard brake, dodge left, and dodge right) is also known. A qualitative analysis of the risk involved while pedestrian and vehicle are in shared space are shown in Tab. 2.1. A high-risk situation means that this is a high risk of collision and low risk defines safe maneuver from the perspective of both vehicle and pedestrian. For example, take the situation when the pedestrian is turning back and the vehicle is dodging right to avoid the collision. Assume the pedestrian is closer to the finish point and decides to turn back suddenly. In such a scenario, there is a possibility of collision because the pedestrian and the vehicle can collide in the left half portion of the lane. The Problem Scenario is created to address several problems. Collision avoidance in case pedestrians and vehicles show sudden erratic behavior, last instant when collision can be avoided, and predicting the accurate position of pedestrian and vehicle at time t at a location where pedestrian and vehicle have a chance of collision. For example, consider a case when a pedestrian decides to suddenly go back and vehicle will have to take collision avoidance measures and also to assess when collision is not avoidable. This analysis will help determine taking safe decisions and low-risk actions to avoid vehicle-pedestrian collisions.

The following novel contributions have been reported in this chapter

- Case Study to avoid vehicle-pedestrian collision
- Accurate prediction of pedestrian and vehicle future position when they have variable velocity
- Decision Matrix for vehicle and pedestrian guidance system for collision avoidance and risk analysis
- General sufficient conditions for vehicle-pedestrian scenarios
- Proof of derived sufficient conditions

2.2 Background

In this section, studies conducted to assess the time gap by pedestrians, contesting shared spaces with pedestrians, and the limitations of studies are discussed.

Authors in [2] defined pedestrian critical gap to be (2.1) where $dist_{pcross}$ is the length of crosswalk or road width. v_p is the speed of the pedestrian and ms is the safety margin.

Table 2.1: Qualitative Analysis of Risk from Vehicle and Pedestrian Viewpoint

Vehicle Action	Pedestrian Action			
	Continue	Stop	Speed up	Turn Back
Slow & Stop	Low Risk	High Risk	Low Risk	Low Risk
Dodge Left	Low Risk	High Risk	High Risk	Low Risk
Dodge Right	Low Risk	High Risk	Low Risk	High Risk
Constant	Low Risk	High Risk	High Risk	High Risk

Any vehicle time gap less than this pedestrian critical gap will result in a collision and correct assessment of the vehicle time gap is essential. In another study [3] data was collected using a video camera in the city center of Athens. Data collected consists of two categories; accepted gap and rejected gap. A logarithmic model was designed to fit the data and finally, model performance is presented. Model elasticity was also evaluated and using the results following correlations were found. Traffic conditions were found to be the most important determinant of pedestrian crossing behavior. A very similar study is also conducted [4]. One noteworthy observation is that females accept longer gap times compared to men. Authors in [5] also collected data from the mid-section pedestrian crossing and found a minimum time gap (3s or 75 ft).

$$gap = dist_{pcross}/V_p + ms \quad (2.1)$$

Another approach [6] collected pedestrian crossing data from a very different region (Hyderabad, India) and also generated a logarithmic model to fit the data. Results show that the gap is correlated with several factors: pedestrian speed, crossing direction, rolling gap, and vehicle speed. In [7], authors used the binary logit technique to model pedestrian crossing behavior using data collected. The model shown is linear and surprisingly accurate in assessing accepted and rejected gaps for the data collected.

The research discussed above is on specific scenarios and even then there is no guarantee of no collision avoidance as statistical models always have some inaccuracy. Furthermore, work done on gap acceptance also points out errors in the gap perception of pedestrians, the higher the speed of the vehicle, the pedestrians tend to accept lower gaps which means pedestrians focus on the distance factor when considering gap which leads us to the conclusion that a guiding system is required to assist pedestrians.

Other approaches include contesting shared spaces between pedestrians and vehicles. In [8] article, authors used a game theoretic approach to decide whether a particular shared space will be occupied by a pedestrian or a vehicle. [9] shows a general method to model a range of pedestrian behaviors and [10] uses machine learning methods to solve the same problem. All approaches mentioned are data dependent and related to contesting shared space during vehicle-pedestrian interaction. Authors in [11] have evaluated very similar work that we are focusing on. They modified the trajectory of the vehicle by predicting the next pedestrian position using the LSTM algorithm. LSTM cannot 100% guarantee correct prediction of pedestrian position and might result in collision secondly vehicle has to be at relatively same speed as the pedestrian to avoid collision. In our work, we aim to solve the problem when vehicle speed is much greater and can assess safe regions and avoid collision.

Although modeling pedestrian gap acceptance behavior can be useful for the decision-making of automated vehicles it can result in accidents as the results do not provide

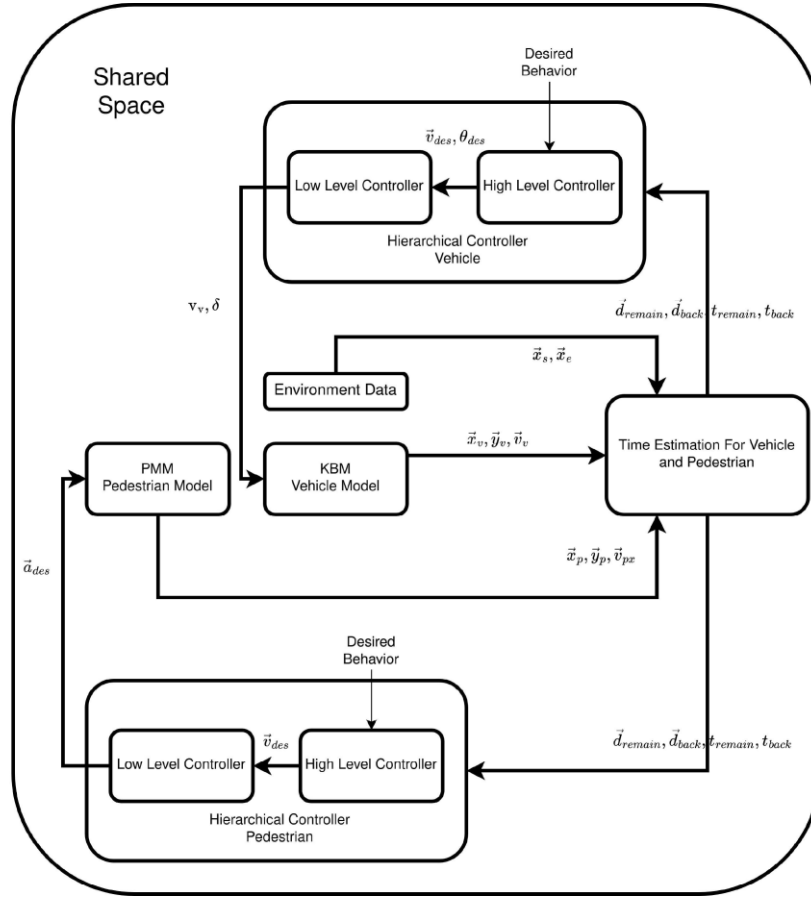


Figure 2.2: System Process

theoretical guarantees. Furthermore, pedestrians tend to have a flawed understanding of the gap when vehicle speed increases. Therefore, this work is focused on providing vehicle and pedestrian guidance systems that will provide a safety guarantee under sufficient conditions considering worst-case behavior from vehicles. The study takes single pedestrian and single vehicle behavior to evaluate critical vehicle and pedestrian time gaps to ensure no collision if sufficient conditions are satisfied. These conditions can then be implemented in handheld devices, mid-section crossings, and vehicle controllers to guide pedestrians and vehicles to cross vehicle-pedestrian shared spaces.

2.3 Methodology

In this section, the overall system, pedestrian and vehicle modeling, and their control, derived sufficient conditions and their proof, and decision matrix for the pedestrian and vehicle guidance systems are presented.

2.3.1 System Process

The system process is shown in Fig. 2.2. The vehicle and pedestrians are controlled through Hierarchical-Finite State Machine (H-FSM) based controllers, a similar method is used in [12]. Vehicle Control System (VCS) receive vehicle and pedestrian location and vehicle

capability and recommend appropriate responses, forward, back, wait, and speed up to pedestrians and appropriate responses, constant speed, brake, hard brake, dodge left, and dodge right to vehicle to avoid collision. This is a decentralized Control problem with two autonomous entities that can make their decisions independently of each other. They make their decisions based on their desired speed and collision avoidance. The vehicle guidance system receive speed and location information using a communication link. Both vehicles and pedestrians have GPS sensors to estimate speed and location.

2.3.2 Vehicle Model

The vehicle is modeled using kinematic bicycle model as shown in equations (2.2), (2.3), (2.4), and (2.34). X_v , Y_v , $[v_{vx} \ v_{vy}]$ and θ_v is the x coordinate, y coordinate, velocity and heading of the vehicle respectively. δ is the steering angle. v_v (2.35) is the speed of the vehicle and L is the length of the bicycle. and acceleration is given by (2.5).

$$\dot{x}_v = v_v \cos(\theta_v) \quad (2.2)$$

$$\dot{y}_v = v_v \sin(\theta_v) \quad (2.3)$$

$$\dot{\theta}_v = \frac{v_v \tan(\delta)}{L} \quad (2.4)$$

$$\dot{\vec{v}}_v = \vec{a}_v = [\vec{a}_{vx} \ \vec{a}_{vy}] \quad (2.5)$$

2.3.3 Pedestrian Model

A pedestrian is modeled as a point mass model as shown in equations (2.6), and (2.7). Where X_p , V_{px} , and a_{ipx} are the position, speed, and acceleration of the pedestrian in the x coordinate system. Since pedestrian position $i]p$ in y coordinate system is constant it is not required to model.

$$\vec{x}_p = \vec{v}_{px} \quad (2.6)$$

$$\vec{v}_{px} = \vec{a}_{px} \quad (2.7)$$

2.3.4 Distance and Time Estimation

Time traveled while the brake is applied to the dynamic object and is approximated as a quadratic equation shown in (2.8). Where $d(to)$, $v(to)$, and $a(to)$ are the magnitude of position, speed, and acceleration of the object respectively at initial time to . And (2.9) is the distance taken by the dynamic object until time t

$$it = \frac{-v(to) + \sqrt{v(to)^2 - 4d(to)a(to)}}{2a(to)} \quad (2.8)$$

$$dt = d(to) + (t - to)v(to) + (t - to)^2 a(to) \quad (2.9)$$

The method described above is shown in [13]. To compensate for errors in transmission and measurements some terms are introduced. It is assumed that the variance of the

measurement error and transmission error is known. Variance in distance because of measurement and transmission error is represented as $\delta.d$ (2.10) where $\delta.dm$ and $\delta.dt$ is the variance of error due to measurement and transmission respectively. Similarly, variance in speed because of measurement and transmission error is represented as $\delta.v$ (2.11) where $\delta.vm$ and $\delta.vt$ is the variance of error due to measurement and transmission respectively. In Fig. 2.1 we have shown some terms $d:ap$ (2.13), $lremain$ (2.14), $lenter$ (2.15), $lcenter$ (2.16), and $lback$ (2.12). Where x_s , X_e , X_c , and x_d are the x coordinate of the starting point, x coordinate of the ending point, lane center, and x coordinate of a point at the lane edge respectively. Collision is defined as (2.17). Where δ_1 is a small constant.

$$\Delta d = \begin{cases} \Delta d_m & \text{if locally available} \\ \Delta d_t + \Delta d_m & \text{if transmitted} \end{cases} \quad (2.10)$$

$$\Delta v = \begin{cases} \Delta v_m & \text{if locally available} \\ \Delta v_t + \Delta v_m & \text{if transmitted} \end{cases} \quad (2.11)$$

$$\vec{d}_{back} = \vec{x}_p - \vec{x}_s + \Delta d \quad (2.12)$$

$$\vec{d}_{gap} = \vec{y}_v - \vec{y}_p + \Delta d \quad (2.13)$$

$$\vec{d}_{remain} = \vec{x}_p - \vec{x}_e + \Delta d \quad (2.14)$$

$$\bar{d}_{enter} = X_s - X_d \quad (2.15)$$

$$\bar{d}_{center} = X_s - X_e \quad (2.16)$$

$$|\vec{y}_v - \vec{y}_p + \Delta d| < \delta_1 \quad (2.17)$$

Distance and Time Estimation for Vehicle

While vehicle distance and time are calculated it is assumed that pedestrian behavior is known and the vehicle adjusts its behavior accordingly to either achieve the goal or to avoid collision. The vehicle is assumed to display three fundamental types of behavior (constant, decelerate, and dodge). Time taken is shown in (2.18). Distance covered by the vehicle is shown in (2.21). For vehicle dodging behavior a fixed change in trajectory length either from the left or right. Fixed change in curve length is represented by a constant $kdodge$ as shown in (2.19).

$$t_{apv} = \begin{cases} ld:apvl / (vv - \delta.v) & \text{constant} \\ tt & \text{decelerate} \\ ld:apvl * / (vv - \delta.v) & \text{dodge} \end{cases} \quad (2.18)$$

$$d:apvl = d:apv + kdodge \quad (2.19)$$

$$\vec{d}_t = d_t \frac{\vec{v}_v}{|\vec{v}_v|} \quad (2.20)$$

$$\vec{d}_{gapv} = \begin{cases} \vec{d}_{gap} & \text{constant} \\ \vec{d}_t & \text{decelerate} \\ \vec{d}_{gapv1} & \text{dodge} \end{cases} \quad (2.21)$$

Time Estimation for Pedestrian

While pedestrian distance and time are calculated it is assumed that vehicle behavior is known and pedestrians adjust their behavior accordingly to either achieve the goal or to avoid collision. Pedestrian is assumed to exhibit three types of behavior (forward, stop, backward). In case the pedestrian is moving forward its time is evaluated in (2.22). Where t_x is the time taken by the pedestrian from the current speed to the desired speed. This value is measured while running experiments. Although, the original work was done to measure the time from a certain speed to come to a stop the same equations can be used if we want to estimate the time from a stop to a certain speed.

$$t_{remain} = t_x + \frac{|\vec{d}_{remain} - \vec{d}_t|}{|\vec{v}_p - \Delta v|} \quad (2.22)$$

In the case, when pedestrian comes to a stop time evaluated is shown in (2.23).

$$i_{stop} = it \quad (2.23)$$

In the case, when the pedestrian goes backward from the middle of the lane time evaluated is shown in (2.24). The time when speed is dynamic is multiplied by a factor of 2 because first the speed will go to zero from v_{pmax} and move $-v_{pmax}$.

$$t_{back} = 2t_x + \frac{l_{ack} - d}{v_p - W} \quad (2.24)$$

In case when pedestrian is at edge of the lane time to enter in a danger zone is evaluated as shown in (2.25). Similarly, t_{center} (2.26) is also evaluated.

$$t_{enter} = i_x + \frac{l_{enter} - d}{v_v - W} \quad (2.25)$$

$$i_{center} = i_x + \frac{i_{center} - d}{v_v - W} \quad (2.26)$$

2.3.5 Pedestrian Region Evaluation

Sufficient conditions are relevant to the Pedestrian Region (**PR**). A state machine is designed as shown in Fig. 2.3. The states define the **PR** relevant to the lane. Four states are defined as Outside Lane (OL), Lane Edge (LE), Left Half Lane (**LHL**), and Right Half Lane (RHL). State transitions are shown in Tab. 2.2.

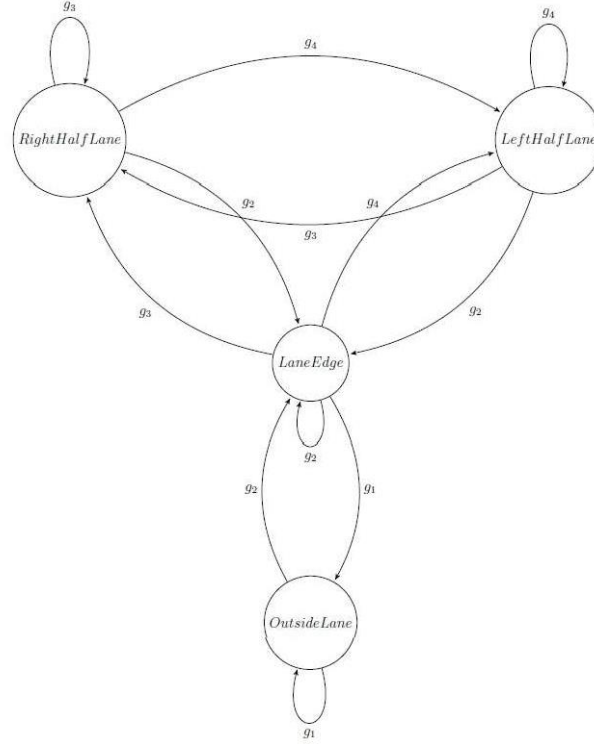


Figure 2.3: State diagram for detecting pedestrian position with respect to road lane

Table 2.2: State Machine Transition Conditions

Transition Condition	
g1	$X_p - X_s < 62 \wedge X_p - X_e < 62$
g2	$a_enter > X_p - X_s > Old_nter \wedge X_p - X_e > 0$
g3	$X_p - X_s > X_enter \ \& \ X_p - X_e < 0$
g4	$X_p - X_s > d_nter \ \& \ X_p - X_e > 0$

2.3.6 Sufficient Conditions (SCs)

As relevant SCs is based on PR relative to the lane. SCs are presented for three separate cases. First case: pedestrian is outside lane. SC required for pedestrian-safe crossing are shown in (2.27). Second Case: Sufficient Condition when pedestrian in at Lane Edge is (2.28). Third Case: when the pedestrian is inside the lane is shown in (2.27) and (2.29). If (2.27) is true pedestrian will simply cross, otherwise if (2.29) holds pedestrian will move back to the starting position to avoid collision. There are certain situations where collision avoidance will not be possible and its details are mentioned in the section Decision Matrices for Vehicle and Pedestrian Guidance Systems.

$$S1 : tremain < tgapv \quad (2.27)$$

$$S2 : tenter > tgapv \quad (2.28)$$

$$S3 : tremain > tgapv \ \& \ tbaek < tgapv \quad (2.29)$$

Table 2.3: Sufficient Conditions for Vehicle Guidance System

Sufficient Conditions	
11	$tremain < tgapv$
12	$tenter > igapv$
13	$tback < igapv$
14	$0req :S 0max$
15	$db < ld apl$
16	$dhb < ld apl$
17	$icenter < igapv$

Table 2.4: Combination of Sufficient Conditions for Vehicle Guidance System I

	Sufficient Conditions
M1	11!
M2	M1 & 12!
M3	M2 & 14!
M4	M3 & 15!
M5	M4 & 16!
M6	13!
M7	M6 & 14!
M8	M7 & 15!
M9	M8 & 16!
M10	14!
M11	M10 & 15!
M12	M11 & 16!

SCs relevant to vehicle control are shown in Tab. 2.3 and Tab. 2.4. where db and dhb are the distances the vehicle moves when smooth and hard deceleration is applied respectively. They are calculated using (2.8) and (2.9) when $-ak$ and $-amax$ is applied respectively using equation (2.5).

2.3.7 Decision Matrix and Analysis for Vehicle Control System

The nomenclature for Vehicle Decision Matrix is shown in Tab. 2.6. Decision Matrices for VCS are shown in Tab. 2.7, Tab. 2.9, and Tab. 2.8.

The Decision Matrices highlight that some situations will cause a collision and no safety measure can be taken if such a situation is encountered. VCS is also capable of making safety avoidance measures before satisfying SCs where collision H is shown in Tab. 2.7, Tab. 2.9, and Tab. 2.8. The combination of relevant sufficient conditions utilized are shown in Tab. 2.5, Tab. 2.4, and Tab. 2.3. Therefore, the goal of the VCS is to make sure such a scenario is not encountered apart from satisfying sufficient conditions. In Tab. 2.7, Tab. 2.9, and Tab. 2.8 it can be observed that in N6, N11, and N14 vehicle will collide with the pedestrians and last safe instant where a collision can be avoided will be the sufficient conditions N5, N10, and N13 while the pedestrian is in forward, back, and wait state respectively.

Table 2.5: Combination of Sufficient Conditions for Vehicle Guidance System II

	Sufficient Conditions
N1	M1 & 12
N2	M2 & 14 & 17
N3	M2 & 14 & 17!
N4	M3 & 15
N5	M4 & 16
N6	M5
N7	M6 & 14 & 17
N8	M6 & 14 & 17!
N9	M7 & 15
N10	M8 & 16
N11	M9
N12	M10 & 15
N13	M11 & 16
N14	M11 & 16!

Table 2.6: Nomenclature for VCS

Command	Symbol
Constant Speed	<i>C</i>
Brake	<i>B</i>
Hard-Brake	<i>HB</i>
Hit	<i>H</i>
Indecisive	<i>I</i>
Dodge Left	<i>DL</i>
Dodge Right	<i>DR</i>

Table 2.7: Decision Matrix for VCS when Pedestrian is in Forward State I

	Combination of Sufficient Conditions						
	11	N1	N2	N3	N4	N5	N6
01	<i>C</i>	<i>C</i>	<i>DL</i>	<i>DR</i>	<i>B</i>	<i>HB</i>	<i>H</i>
LE	<i>C</i>	<i>C</i>	<i>DL</i>	<i>DR</i>	<i>B</i>	<i>HB</i>	<i>H</i>
RHL	<i>C</i>	<i>I</i>	<i>DL</i>	<i>DR</i>	<i>B</i>	<i>HB</i>	<i>H</i>
LHL	<i>C</i>	<i>I</i>	<i>DR</i>	<i>DR</i>	<i>B</i>	<i>HB</i>	<i>H</i>

2.3.8 Hierarchical Controller for Vehicle

The hierarchical Controller for Vehicle (HCV) is shown in Fig. 2.2. Finite State Machine for Vehicle (FSMV) for High-Level Controller is shown in Fig. 2.4. FSMV has five states Constant, Brake, Hard Brake, Dodge Left, and Dodge Right. State transitions are shown in Tab. 2.10. For the low-level controller, Stanley controller is used as shown (2.30), where k_s are constants and $esteer$ (2.31) is the angular error between current heading θ_v and the required heading angle θ_{des} to follow the trajectory. The vehicle acceleration is controlled through a low-level controller based on the Proportional Integral (PI) controller shown in (2.32). where K_p and k_i are scalars and $e = [e_x, e_y]^T$ is shown in (2.33). where v_{dx} , and

Table 2.8: Decision Matrix for VCS when Pedestrian is in BACK State II

	Combination of Sufficient Conditions					
	L3	N7	N8	N9	N10	N11
OL	C	C	C	C	C	C
LE	C	C	C	C	C	C
RHL	C	DL	DR	B	HB	H
LHL	C	DR	DR	B	HB	H

Table 2.9: Decision Matrix for VCS when Pedestrian is in Wait State III

	Combination of Sufficient Conditions				
	L4 & L7	L4 & L7!	N12	N13	N14
OL	C	C	C	C	C
LE	C	C	C	C	C
RHL	DL	DL	B	HB	H
LHL	DR	DR	B	HB	H

v_{dy} is the desired velocity taken from a predefined vehicle trajectory.

$$\delta = \delta_{des} + \frac{-1}{\tan \delta} k_{steer} \frac{v_{dy}}{v_v} \quad (2.30)$$

$$e_{steer} = \theta_{des} - \theta_c \quad (2.31)$$

$$\vec{a}_v = \begin{bmatrix} \vec{a}_{vx} \\ \vec{a}_{vy} \end{bmatrix} = k_p \begin{bmatrix} \vec{e}_x \\ \vec{e}_y \end{bmatrix} + k_i \int \begin{bmatrix} \vec{e}_x \\ \vec{e}_y \end{bmatrix} dt \quad (2.32)$$

$$e = [e_x \ e_y] = \begin{bmatrix} d_x - v_x \\ v_{dy} - v_y \end{bmatrix} \quad (2.33)$$

$$\vec{v}_v = [\vec{v}_{vx} \ \vec{v}_{vy}] = \int \vec{a}_v^T dt \quad (2.34)$$

$$v_v = \|\vec{v}_v\| \quad (2.35)$$

Table 2.10: Transition Conditions for FSMV

Symbol	Transition Condition
tc1	L1!L2!L3
tc2	L2!&L1!&L3!&L4
tc3	L2!&L1!&L3!&L4!&L5
tc4	(L1!&L2!&L3!&L4!&L5!&L6!)&(g3)
tc5	(L1!&L2!&L3!&L4!&L5!&L6!)&(g4)

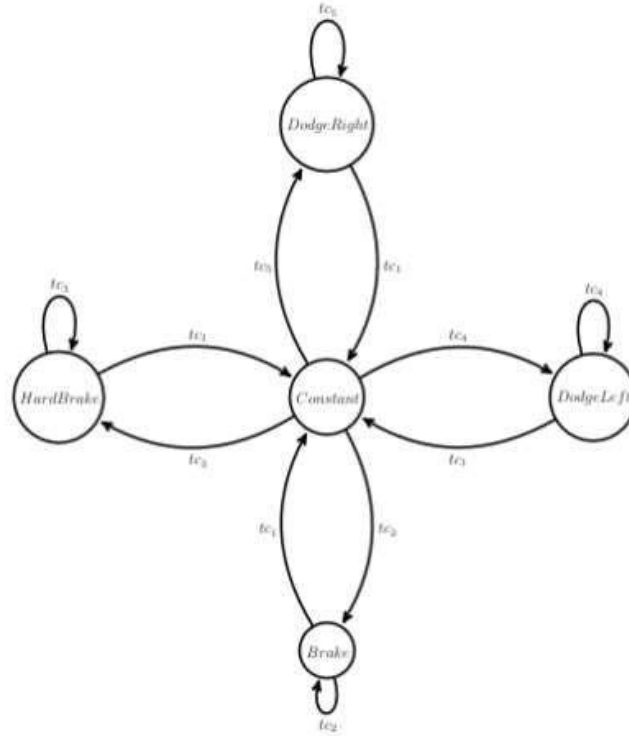


Figure 2.4: Finite State Machine for Vehicle High Level Controller

2.4 Simulation Setup, Results, and Discussion

The simulation Environment used to show the effectiveness of the method is shown in Fig. 2.5. The red rectangle is the vehicle small the red circle is the pedestrian. The triangles are starting and ending locations. Red lines are lane boundaries and the blue dotted line is the trajectory. The modeling used for a pedestrian and a vehicle are described in section 2.3.3, and 2.3.2 respectively. The Pedestrian and the vehicle have a mass M of 10 and 30 Kg respectively. The pedestrian has a maximum speed and maximum acceleration of 0.5 m/s , and 0.1 m/s^2 , similarly vehicle has a maximum speed and acceleration of 8 m/s , and 3 m/s^2 . given these values db and dhb for the vehicle are evaluated as 4.3 meters and 3.3 meters respectively when -1 and -2 deceleration is applied. and for pedestrians. The time to reach from 0 to 0.5 m/s is 0.08 seconds and from 0.5 to -0.5 is 0.16 seconds. Similarly, the distances covered are 0.03 meters and 0.077 meters respectively. The results for two case studies are presented.

Case I

In this scenario, the environment used shows the effectiveness of the VCS and it is assumed that pedestrian behavior is known. The pedestrian is placed again in the middle of the road but is in the wait state. The vehicle starts from 30 meters in the y direction and has a constant speed of 8 m/s . Since sufficient condition, L4 Tab. 2.2 satisfies Command issued by VCS to the vehicle is Brake with deceleration -1 m/s^2 and the vehicle starts decelerating to stop the collision. Results are shown in Fig. 2.6. The blue curve is the gap maintained by the vehicle and then the vehicle starts decelerating and comes to a halt

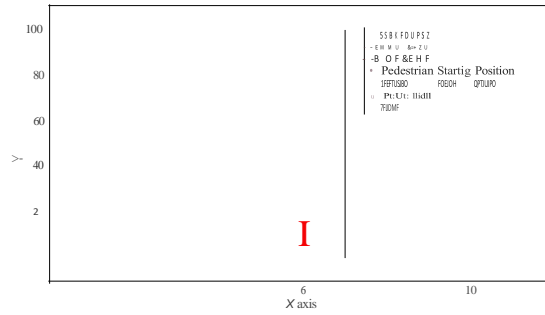


Figure 2.5: Simulation Setup

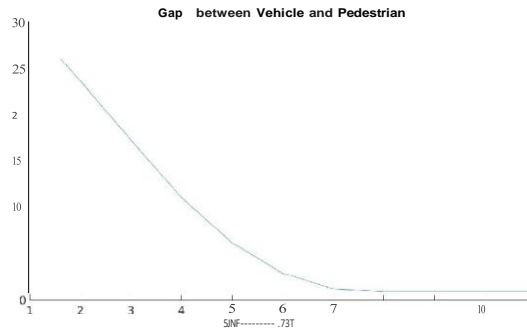


Figure 2.6: Gap Between Pedestrian and Vehicle Case I

when the gap is approximately 2 meters verifying the methodology.

Case II

In this scenario, the environment used shows the effectiveness of the VCS dodging the pedestrian to avoid a collision. The pedestrian starts from the starting position with a speed of 0.3m/s and continues moving forward. Transition Condition tc_4 in Fig. 2.4, Tab. 2.10 and vehicle dodge left to avoid the collision. the vehicle moves with a constant speed of 8m/s and the trajectory is modified using a Gaussian function as shown in (2.36). The results are shown in Fig. 2.7 and Fig. 2.8. In Fig. 2.7 trajectory generated (blue) and the trajectory followed (green) by the vehicle is shown while the yellow line is the trajectory of the pedestrian. In Fig. 2.8 gap vs pedestrian region is shown. It can be observed that when the pedestrian is in the right half Lane vehicle is crossing since the gap is closer to zero and when the pedestrian reaches the left half of the Lane vehicle has already crossed signifying a gap of more than 15 meters.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} 2e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (2.36)$$

2.5 Conclusion

Pedestrians are prone to accidents and make flawed decisions based on their understanding of critical gaps in a vehicle-pedestrian shared space. To solve this problem a conventional

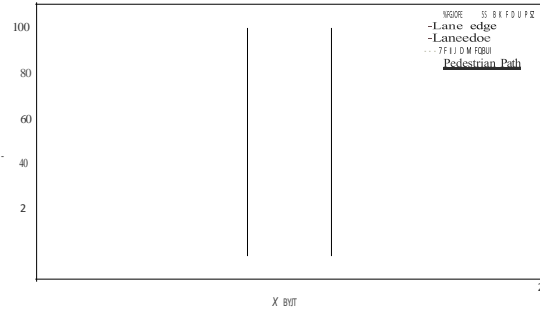


Figure 2.7: Trajectory of a vehicle while dodging left Case II

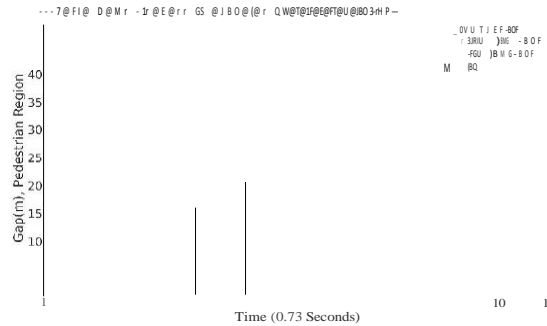


Figure 2.8: Gap Between Pedestrian and Vehicle Case III

approach is developed. This study focuses on providing theoretical guarantees under a range of pedestrian and vehicle behaviors for safe vehicle and pedestrian crossings, making the appropriate safe decision to avoid collisions, and evaluating limits of collision avoidance capability. The study covers cases when pedestrians make sudden changes in their behavior (Back, Wait) and appropriate decision-making (decelerate, dodge) to avoid a collision. Decision Matrices are provided to guide pedestrians in case vehicle behavior changes while a pedestrian is in a shared space and vice versa. Simulation results show that the proposed method is effective. The solution proposed is scalable as the required infrastructure is already in place because automated vehicles and most pedestrians carry handheld devices with communication capability and location detection sensors. Automated vehicles and pedestrians already carry GPS sensors and communication capability. Future work involves an extension of current work to multi-lanes and intersection crossings.

Chapter 3

FSM-based Control of Automated Vehicles in presence of an Indecisive Pedestrian

This chapter is derived from the work yet to be published.

3.1 Introduction

Recent accidents involving pedestrians and Autonomous Vehicles (AVs) raise critical questions about safety, even when pedestrian actions contribute to the incident. While some may argue that pedestrians bear the primary responsibility in certain cases, the potential for AVs to take preventive measures should still be explored.

Some of the case studies are presented below to highlight the need for safety critical vehicle control. Arizona Accident: A pedestrian crossing a highway at night without lights, while dragging a bicycle, was misclassified as an object by the AV, resulting in a collision [14].

GM's Cruise accident: A pedestrian crossed against a red light and was struck by a manually driven vehicle. Thrown into an adjacent lane, the pedestrian was nearly struck again by a following AV from GM Cruise, which barely stopped in time and then dragged the pedestrian to the side of the road. These examples demonstrate the limitations of current AV technology and the need for improved pedestrian detection and response systems [15].

Waymo Dataset: Data from Waymo, a self-driving car company, suggests that a significant portion of collisions involving AVs occur due to unpredictable pedestrian behavior [16].

These incidents underscore the importance of designing control systems for AVs that can mitigate collisions even in the face of unexpected pedestrian actions.

3.1.1 Simulation Case Study

This study investigates a simulated environment (shown in Fig. 3.1) designed to evaluate control strategies for autonomous vehicles (AVs) in avoiding collisions with pedestrians. The AV can perform actions like swerving (dodge), braking, and emergency braking (hard brake) while navigating longitudinally through the road. The pedestrian, with limited

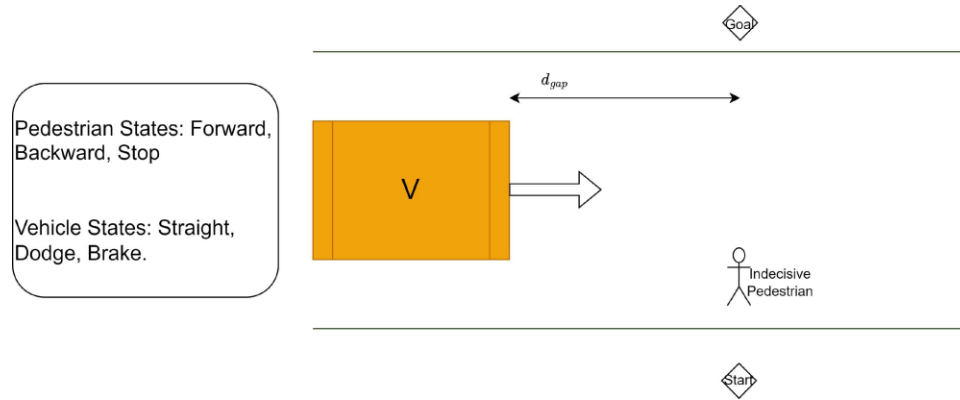


Figure 3.1: Problem Scenario.

movement options (forward, backward, and stop), can also change its goal and direction unexpectedly. The key variable is the initial distance between the vehicle and the pedestrian, denoted as d_{gap} . The primary goal of this research is to develop a control strategy for AVs that can effectively prevent collisions with pedestrians, even in situations where the pedestrian's intentions are unclear.

The following outline the unique contributions of this study are the following

- A physics based control strategy to avoid collision and optimal control
- Comparison of designed control strategy with statistical controller

3.2 Summary of Methodology

Figure 3.2 illustrates the overall system process. The autonomous vehicle (AV) is equipped with LiDAR (Light Detection and Ranging), GPS (Global Positioning System), and IMU (Inertial Measurement Unit) sensors. The vehicle utilizes its own approximate model and fuses this information with GPS data to achieve more accurate positioning. Pedestrian location data is detected using LiDAR. After filtering and processing this information, the AV calculates the time gap between itself and the pedestrian.

Based on the calculated time gap, the AV employs a hierarchical control system to devise a collision avoidance strategy. This may involve actions like braking, swerving, or maintaining course depending on the situation. The pedestrian is modeled with a vision system that estimates the distance and speed of the approaching vehicle. This information is then used to make a decision about crossing the road. Additionally, the pedestrian is modeled with a Finite State Machine (FSM) controller that randomly switches between different pedestrian states (e.g., waiting, crossing, etc.). This randomness makes it impossible for the AV to perfectly predict the pedestrian's goal and next move. A secondary controller will be developed in future work. This controller will attempt to estimate the pedestrian's intention based on available data and use that information to devise a control strategy. The performance of this intention-aware controller will then be compared to the current strategy that relies solely on time gap information.

Figure 3.3 depicts a critical element of the control strategy design. Here, a two-lane road is divided into five distinct regions based on a pre-defined design specification. Notably, the "pedestrian unknown region" should not exceed half the width of a lane accord-

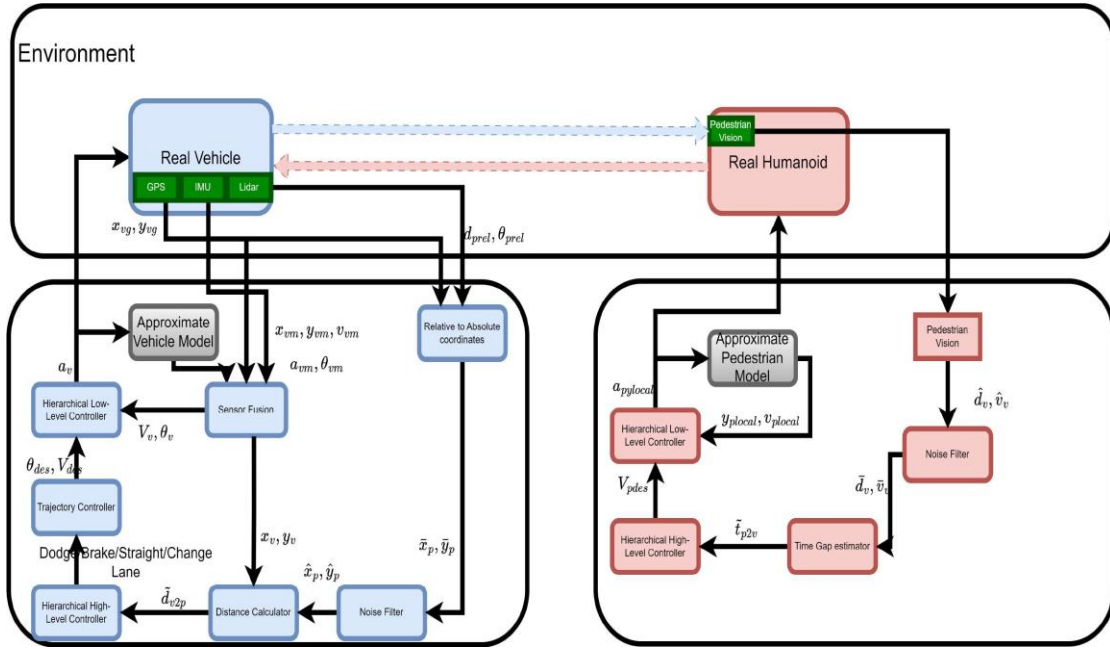


Figure 3.2: Problem Scenario.

Table 3.1: Decision Matrix to avoid collision with pedestrian

Pedestrian Region	Vehicle Decision				
	Dodge Left	Dodge Right	Lane Change	Brake	Hard Brake
1	True	False	False	True	True
2	False	False	True	True	True
3	False	True	False	True	True

ing to this specification. Leveraging this defined region and the pedestrian's maximum speed, we can estimate a time range for the pedestrian's potential movement. This time range is then used to assess the pedestrian's most likely location within that timeframe. Based on this assessment, the AV's control system can determine the appropriate action to take (e.g., swerving, braking, maintaining speed, or slowing down) to prevent a collision. Following the definition of these regions, a decision matrix is generated as shown in Table 3.1. This Decision Matrix detail the specific actions the AV's control system should take based on the pedestrian's location within each region.

3.2.1 Outcomes

This study aims to develop a physics-based control method for autonomous vehicles (AVs) that prioritizes collision avoidance. We will then evaluate the performance of this method compared to control strategies that rely on pedestrian intention estimation.

The primary objective is to determine whether a physics-based control approach surpasses other prediction methods in terms of ensuring pedestrian safety in autonomous vehicle operation.

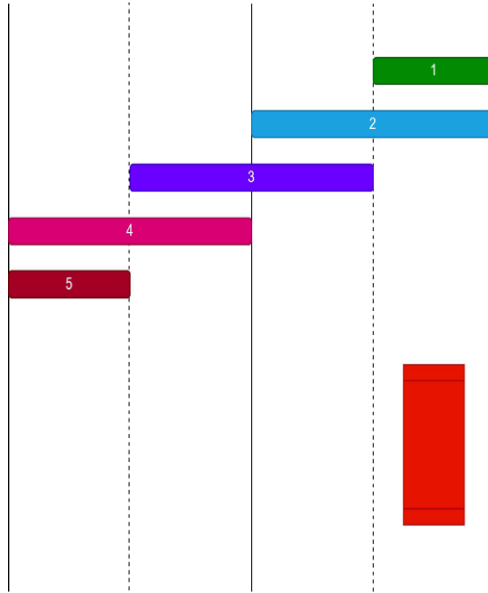


Figure 3.3: Problem Scenario.

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APPENDIX

Autonomous Vehicle and Pedestrian Collision Avoidance From a Deep Reinforcement Learning Viewpoint

Below is a description of the current achieved goal where the vehicle is an intelligent DRL agent. The ultimate goal is to make both the vehicle and pedestrian intelligent agents. This work is ongoing.

A1. Introduction

Navigating pedestrian-populated environments is a challenging task for autonomous vehicles (AVs). The uncertainty introduced by pedestrians and unpredictable road users adds complexity to the AV's decision-making process. Addressing this challenge is crucial to ensuring both safety and efficiency. Recent advancements in artificial intelligence, particularly Deep Reinforcement Learning (DRL), have shown promising results for solving AV-pedestrian collision problems [1-4]. Research has extended to handling pedestrian crossing uncertainty and interaction dynamics between AVs and pedestrians [5-7]. This work describes developing a DRL-based collision avoidance system for AVs, focusing on a scenario where a vehicle must avoid a pedestrian at a crossing.

A2. Addressed Scenario

The problem addressed in this work involves a typical pedestrian crossing scenario, as illustrated in Fig. A1. Here, the AV moves towards a crossing line. In contrast, a pedestrian crosses the road perpendicularly. The AV must take real-time actions to avoid colliding with the pedestrian while maintaining efficient progress. The challenge is balancing safety and speed, ensuring the AV avoids unnecessary braking while preventing collisions.

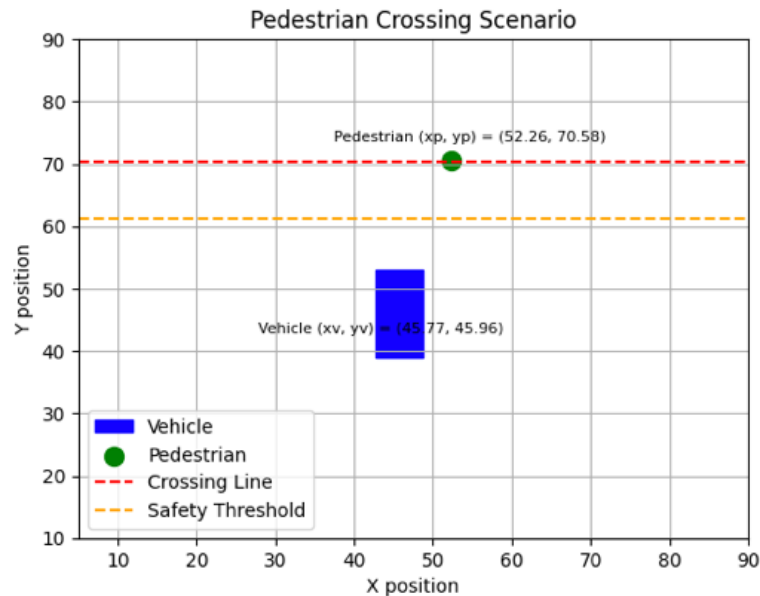


Fig. A1. Addressed Scenario

A3. Translating to DRL World

The described scenario is translated into a DRL setup. The AV is modeled as an intelligent agent in a simulated environment, where it continuously receives the state of the environment, including the positions and velocities of both the vehicle and the pedestrian. The AV can take five actions: driving straight (0), braking (1), dodging right (2), and dodging left within lane (3) and beyond lane (4). The agent aims to navigate the crossing without colliding with the pedestrian while optimizing its speed and progress through a structured reward function.

A3.1. Reward Function

In this system, collision avoidance is enforced through a sophisticated reward structure. The main components of the reward function are:

- **Safe distance reward:** This reward encourages the AV to maintain a distance of at least 5 units from the pedestrian, with higher rewards for greater distances.
- **Collision penalty:** The AV is penalized heavily for any collision risk with the pedestrian. If the pedestrian is within 5 units of the AV and a collision is imminent based on future velocity predictions, the AV is penalized up to 10 points.
- **Progress reward:** The AV is rewarded for moving closer to the crossing line, encouraging continuous progress rather than unnecessary stops.
- **Speed reward:** Maintains an optimal speed range between 4 and 5 units, rewarding the AV for staying within this safe speed range while penalizing slower or excessively fast driving.

The system's collision logic is based on predicting the relative positions of the AV and pedestrian over time. The risk of collision is computed by predicting future positions and velocities. If the expected distance between the AV and the pedestrian becomes dangerously small, the collision risk increases, and the AV is penalized. This logic ensures that the AV takes appropriate actions before a collision occurs.

A3.2 Proximal Policy Optimization (PPO) Implementation

We use a Proximal Policy Optimization (PPO) method to train the AV's decision-making policy. PPO is chosen for its robustness and stability in handling such controlling tasks. The vehicle's policy=actor network outputs a categorical distribution over the discrete action space, while the value=critic network estimates the expected future reward. The actor is trained iteratively, using mini-batch updates and clipped probability ratios to prevent drastic updates. The PPO method ensures that the AV's policy evolves smoothly over time. The training involves:

- Running the AV through multiple episodes in the environment.
- Collecting state-action-reward transitions.
- Using this data to update the policy.

The system uses a replay buffer to store experiences, with prioritized experience replay focusing on important transitions. This enhances the system's learning efficiency, allowing it to focus on higher-risk scenarios and more valuable learning opportunities.

A4. Preliminary Results

A4.1 Training Performance

The vehicle agent is trained over 40,000 frames, with PPO updates performed periodically using mini-batches sampled from the replay buffer. The results demonstrate effective learning, with the agent achieving increasingly higher accumulated and average rewards and lower collision rates as training progressed. Fig.A2 illustrates a sample of accumulated and average rewards over the training iterations. The average reward over 100 episodes reached a stable value, indicating the policy's convergence.

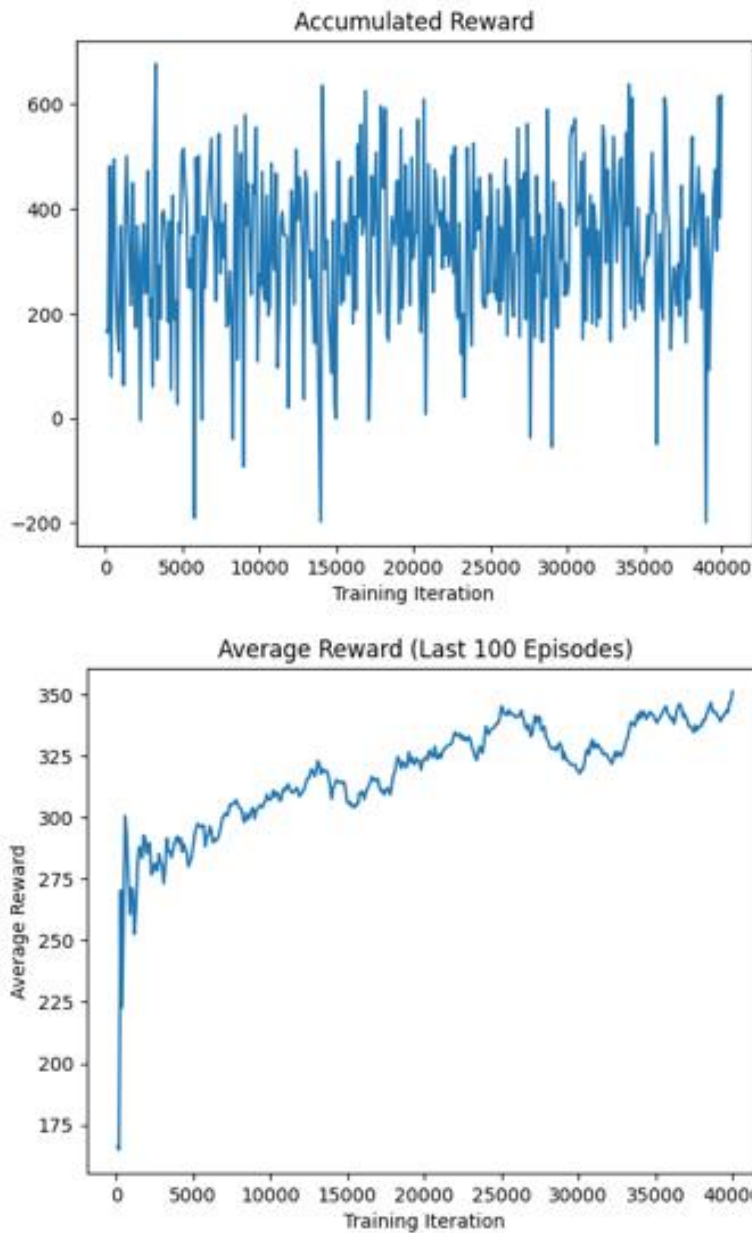


Fig. A2. Training's Accumulated and Averaged Reward

A4.2 Collision Avoidance and Action Analysis

Analysis of the collision rates and action distribution, Fig. A3, shows that the vehicle the agent learns to avoid collisions effectively. The success rate for collision-free episodes reached 99%, in line with results from other DRL-based pedestrian avoidance studies. The vehicle in this sample chooses to drive straight or dodge right or left outside lane when approaching the pedestrian at the crossing line, depending on the pedestrian's position. This behavior mimics human driving patterns in similar real-world scenarios. Additionally, braking was used minimally, only when necessary to avoid imminent collisions.

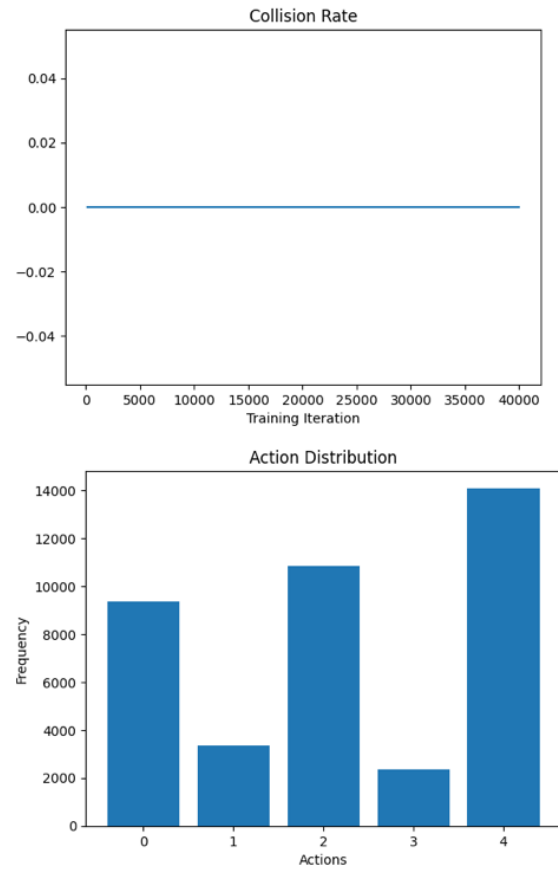


Fig. A3. Training's Collisions Rates and Actions Distribution

A5. Conclusion and Future Work

The PPO-based DRL agent successfully navigated the pedestrian crossing scenario, balancing collision avoidance with minimal speed reduction. The designed reward structure effectively encouraged the vehicle to take necessary actions only when required, avoiding unnecessary braking and dangerous collisions. A key direction for future steps involves modeling indecisive pedestrian behavior. By extending the system to a multi-agent setup where both the pedestrian and vehicle are treated as independent agents, we aim to capture more complex, real-world interactions. This multi-agent scene will allow us to model unpredictable pedestrian movements, such as sudden stops or changes in direction, requiring the vehicle to adapt its strategy dynamically. Such scenarios are increasingly important as autonomous driving technologies become more prevalent.

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