Aggregating Traffic Volumes Estimated from Video Imagery Collected on Repeated Bus Passes: Empirical Evaluation of Different Approaches

Thesis

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By

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Abstract

Roadway segment traffic volumes provide valuable information for a wide spectrum of transportation planning, design, and operation purposes. To expand spatial-temporal coverage of data collection for estimating traffic volumes, it has been proposed to use video data from transit buses. Transit buses are already covering the major roadways in an urban network. Many buses already have cameras deployed for other purposes, but which observe vehicles on a roadway segment that can conceivably be used to estimate traffic volumes. However, unlike traditional techniques that count traffic volumes at a fixed location on a roadway, this moving observer platform (the bus) and sensor (the video camera) observe vehicles at different points along roadway segments at different times. A method has previously been developed to convert these observations to the equivalent of a traffic volume passing a fixed location on the roadway. The volumes estimated in this way are of short duration, however, and therefore are likely to give an unrepresentative estimate of traffic volumes over time periods of interest. To get a good estimate of traffic volume for an estimation period, the multiple volume estimates obtained from the individual passes within that period can be aggregated. This thesis proposes and evaluates different methods to aggregate the volumes derived from individual bus passes.

Different aggregation approaches, which are categorized into “discrete time interval” and “continuous time’’ approaches, are presented. The first category, representing

the first general approach, aggregates individual bus pass volumes by taking their arithmetic average in discrete time intervals. The second category considers bus passes as flow rates that can be integrated over continuous time to determine a volume estimate in a specified time interval. In one implementation of this continuous time category, considered the second general approach, a benchmark interpolation of the individual bus pass flow rates is used to determine the flow rate function. A different implementation of continuous time aggregation, which is considered the third general approach, attempts to smooth the flow rate function by using a "moving median time window" technique before integrating the rates over time.

Evaluation of the approaches is conducted using empirical video data collected from Campus Area Bus Service buses serving The Ohio State University and concurrently collected road tube counts provided by the Mid-Ohio Regional Planning Commission. Volumes that have been previously estimated from the individual bus passes are used as one set of data (Data Variant 1) that serves as input to each of the three approaches. A second set of data (Data Variant 2) used as input to each of the three approaches consists of volumes from individual bus passes that have been transformed through a regression model. Various regression specifications are developed and evaluated, and one is chosen for use in the regression-based transformation of individual bus passes.

The combination of the three approaches and two Data Variants leads to six "Methods" whose estimated volumes are compared to traffic volumes obtained from the concurrently collected road tube data using several metrics. The evaluation results indicate that the aggregation Methods based on integrating the flow rate function systematically improve the volume estimates compared to aggregation based on arithmetic averaging. The Methods that smooth the flow rate function with the moving median time window lead to results that differ from those obtained with the benchmark flow rate function. For some comparisons the moving median approach performs better, while for other comparisons the moving median approach performs worse. It is noted that only one set of smoothing parameters (namely a 30-minute time window with a 1-minute step size) is used in this study. Therefore, having several comparisons showing that smoothing with these somewhat arbitrarily chosen parameters improves performance is encouraging that further research could lead to a better smoothing approach to be used when estimating volume by integrating the flow rate function.

Although not the main focus of this thesis, other consistent results are seen in the empirical evaluations. Specifically, regardless of the Method: there is no evidence of systematic over- or underestimation of period volumes when using the bus video data; lower relative errors are obtained for longer (specifically, 60-minute) estimation periods than for shorter (specifically, 15-minute) estimation periods; and transforming the individual bus pass volumes through a regression model calibrated on an independent data set leads to better traffic volume estimates.

Dedication

This thesis is dedicated to my husband, Nariman Laal Dehghani, my father, Afshin Charmchi Toosi, my mother, Fariba Baghbani, and my brother, Shahin Charmchi Toosi

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Chapter 1: Introduction

1.1. Background and Motivation

Roadway segment traffic volumes are useful for many transportation planning, design, and operation purposes. For example, traffic volumes are used to determine the level-of-service of roadway segments as a means to assess the adequacy of the transportation facilities (see, e.g., National Research Council, 2000). Traffic volumes are used to determine signal timing or to evaluate delays at signalized intersections (see, e.g., Mannering and Washburn, 2016; McCord et al., 2020). Accident analyses require traffic volumes to determine accident rates from raw accident counts. Traffic volumes on a segment are used to calculate vehicle distance travelled (see, e.g., Roess et al., 2004), which is the primary summary measure of travel across a transportation network and often used to portray trends in motorized travel over time.

Traffic volume is the number of vehicles that pass a point on the roadway in a specified time interval and is traditionally estimated by traffic counts – either obtained from sensors such as loop detectors or road tubes, or from human observers – at a fixed location on a road. The typical approach records vehicles that pass the fixed point on the roadway over an extended time and directly provides volumes for intervals that are usually shorter than the time over which the counts are taken. The traffic counting techniques are well developed and are, therefore, very useful for the locations where they are used. However, because of limited number of sensors or human counters, fixed-location sensors can only be deployed at a subset of the segments at any one time. Data are also not collected, or collected very infrequently, on most of the roadway segments. Therefore, it is costly to implement the fixed-location counters and collect the counts across entire network.

Vehicles that travel multiple roadway segments can serve as “moving observers” to detect traffic with more extensive spatial coverage than fixed location sensors. Unlike traditional counts, the moving observer traverses the segment in a very short time and, therefore, likely provides an unrepresentative sample of conditions over longer time periods. However, if the moving observer traverses the segments many times, multiple samples of traffic conditions can be obtained, and aggregating these samples could potentially lead to good volume estimates.

Transit buses that serve urban areas can be considered as potential moving observer platforms (McCord et al., 2020). The presence of buses makes the platform already available, and many of the buses are also equipped with video cameras for other purposes, such as safety, security, and liability. Since the video cameras are already being used on transit buses, no additional cost would be needed for establishing the sensors. Additionally, since buses pass the same roadways on a repeated basis, they can provide estimates that can be aggregated to estimate traffic volumes within time-of-day intervals of interest.

As presented in McCord et al. (2020), there are two steps to estimate a volume during a specified period from the observations obtained from repeated passes of transit buses over a roadway segment. First, the vehicle observations from the traversal of a bus over the segment (what will be called a “bus pass”) are processed into a volume estimate.

This approach is not the focus of this thesis, but it will be briefly reviewed in Section 2.3 and accepted as the starting point for the analyses conducted and presented in subsequent chapters.

The second step, which is the focus of this research, involves aggregating the multiple volume estimates obtained from the individual bus passes to determine a volume for an interval of time. There could be several ways to aggregate the bus pass-derived volumes. Three approaches are considered in this thesis, one of which is the basic approach used in McCord et al. (2020). Each approach will be applied to two “Data Variants” of empirically collected data for a total of six (three approaches × two Data Variants) “Methods.” These aggregation Methods, which will be discussed in subsequent chapters, are categorized into “discrete” and “continuous” time interval approaches. Two of the Methods (one approach with the two Data Variants) are considered discrete time interval approaches; the other four Methods (two approaches with two Data Variants each) are considered continuous time interval approaches. These approaches, Methods and Data Variants are summarized in Table 1.1. The accuracy of these approaches and Methods will be evaluated individually.

Table 1.1: Summary of Approaches, Data Variants, and Methods

Time Interval	Approach	Data Variant 1	Data Variant 2
		Directly Obtained from Bus Passes	Bus Passes after Regression-based Transformation
Discrete	Arithmetic Average	Method 1	Method 2
Continuous	Integral-Benchmark interpolation	Method 3	Method 4
	Smoothed interpolation	Method 5	Method 6

1.2. Objectives and Scopes

The objective of this study is to investigate different approaches to aggregate volume estimates obtained from individual bus passes into an estimate for a time interval of specific duration, namely, 15- and 60- minute estimation intervals. Three approaches are considered and evaluated using volumes obtained from bus video data and road tube data. Two variants of the empirical data are considered. One variant considers aggregation of video-based volume estimates directly obtained from the individual bus passes. The second variant considers aggregation of the bus pass volumes after transformation through a regression model. Therefore, using these two variants and three approaches leads to six aggregation Methods which are as follows:

- Method 1: Arithmetic averaging of volume estimates directly obtained from bus passes
- Method 2: Arithmetic averaging of volume estimates from bus passes after regression-based transformation
- Method 3: Integral-benchmark approach with flow rates determined directly from individual bus passes
- Method 4: Integral-benchmark approach with flow rates from individual bus passes after regression-based transformation
- Method 5: Smoothed curve approach with flow rates determined directly from individual bus passes
- Method 6: Smoothed curve approach with flow rates from individual bus passes after regression-based transformation

Empirical evaluation of the accuracy of these Methods is conducted using video data collected from The Ohio State University Campus Area Bus Service buses and concurrently collected road tube vehicle counts provided by the Mid-Ohio Regional Planning Commission.

1.3. Thesis Organization

This thesis is organized in five chapters. In this first chapter, the background and motivation of the study are presented. In the second chapter, the network and data that are used in the empirical study are discussed. The method presented in McCord et al., (2020) to estimate a traffic volume from the observations in a single bus pass is also summarized. Chapter 3 consists of two somewhat separate sections. In Section 3.1, the estimation periods of interest in this study are presented, the evaluation metrics to quantify the accuracy of the estimates are specified, and three ways (what will be called “Cases”) are defined to handle individual bus pass volumes that are considered exceptionally low or high. Section 3.2 presents an approach to aggregate volumes obtained from individual bus passes based on the arithmetic average of the single pass volumes in discrete time intervals. The results obtained from this approach when applied to directly obtained bus pass volumes (Method 1) are presented in Section 3.2.1 and when applied to the bus pass volumes transformed through a regression model (Method 2) are presented in Section 3.2.2. Chapter 4 presents and evaluates an approach to aggregation considering continuous time. In Section 4.1, the concept of determining volumes by integration of flow rates is presented. In Section 4.2, Method 3 (considering bus pass volumes directly obtained from individual bus passes) and Method 4 (considering bus pass volumes after being transformed with a regression model), which use the integration approach based on a benchmark implementation, are discussed. In Section 4.3, the integration approach is again considered but based on a “smoothing” of the flow rates in time. Sections 4.3.2 and 4.3.3, respectively, present Method 5 (considering bus pass volumes directly obtained from individual bus passes) and Method 6 (considering the transformed bus pass volumes through a regression model), which use the smoothing approach based on a moving median time window implementation. Finally, in chapter 5, findings obtained from this study and conclusions are summarized, and recommendations for future research are made.

Chapter 2: Background – Empirical Setting and Estimation of Single Pass Video Volumes

2.1. Network Used in Empirical Study

The Ohio State University (OSU) Columbus campus is one of the largest single campuses worldwide and comprises diverse land uses with an area of 1,600 acres and nearly 100,000 students, faculty, and staff (OSU Paths with Purpose, 2021). The campus has a transportation network that serves commuting traffic to and from the campus, local traffic within the campus, and Columbus area traffic through the campus. In this study, the major streets in the campus roadway network and vehicular travel on these roadways are used as the setting for the empirical data. More specifically, two subsets of the campus networks are selected for analysis in this thesis and described as follows:

- A “2018 network”, which consists of 21 bi-directional roadway segments (i.e., 42 directional roadway segments) that total 6.26 directional roadway miles. The 2018 network is depicted in Figure 2.1.
- A “2019 network”, which consists of 24 bi-directional roadway segments (i.e., 48 directional roadway segments) that total 7.86 directional roadway miles. The 2019 network, which is depicted in Figure 2.2, is a superset of the 2018 network and contains four additional segments.

In Figures 2.1 and 2.2, segments without numbers are segments that are used for purposes beyond the scope of this study. Segments outlined in red are those where road tube data were collected (see Section 2.2.2) and are used in the empirical analyses conducted in Chapters 3 and 4. As seen in the figures, the segments are primarily either northbound/southbound or eastbound/westbound segments. Segments are numbered as $S.s$, where S denotes the segment's number, and s indicates the segment's direction. To specify the direction, s is selected as 1 for a northbound or eastbound direction and 2 for a southbound or westbound direction. A segment that is part of more than one year's network has the same number $S.s$ in the two (2018 and 2019) networks.

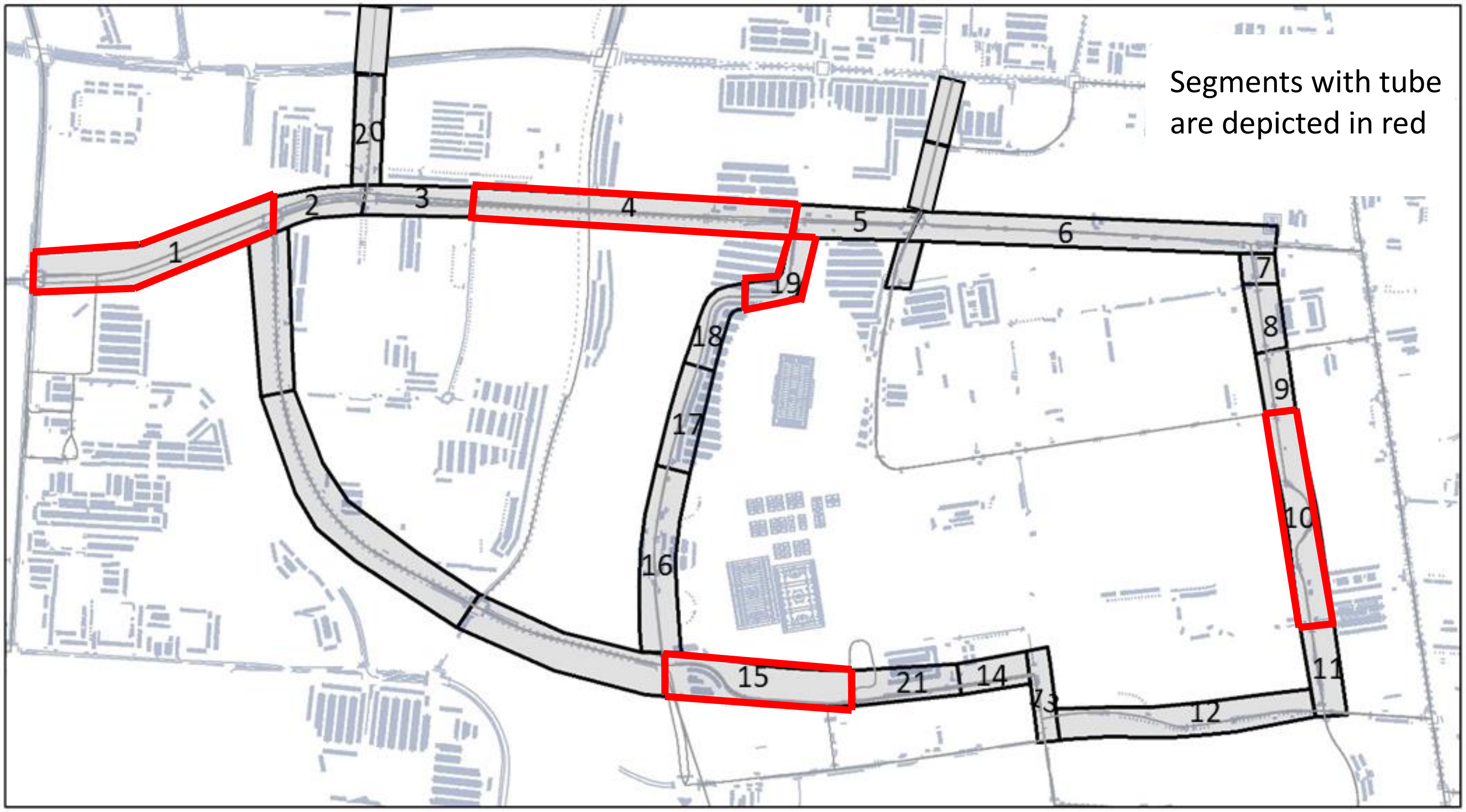

Figure 2.1: OSU campus roadway network used in 2018 data collection: red segments are those where road tubes were placed

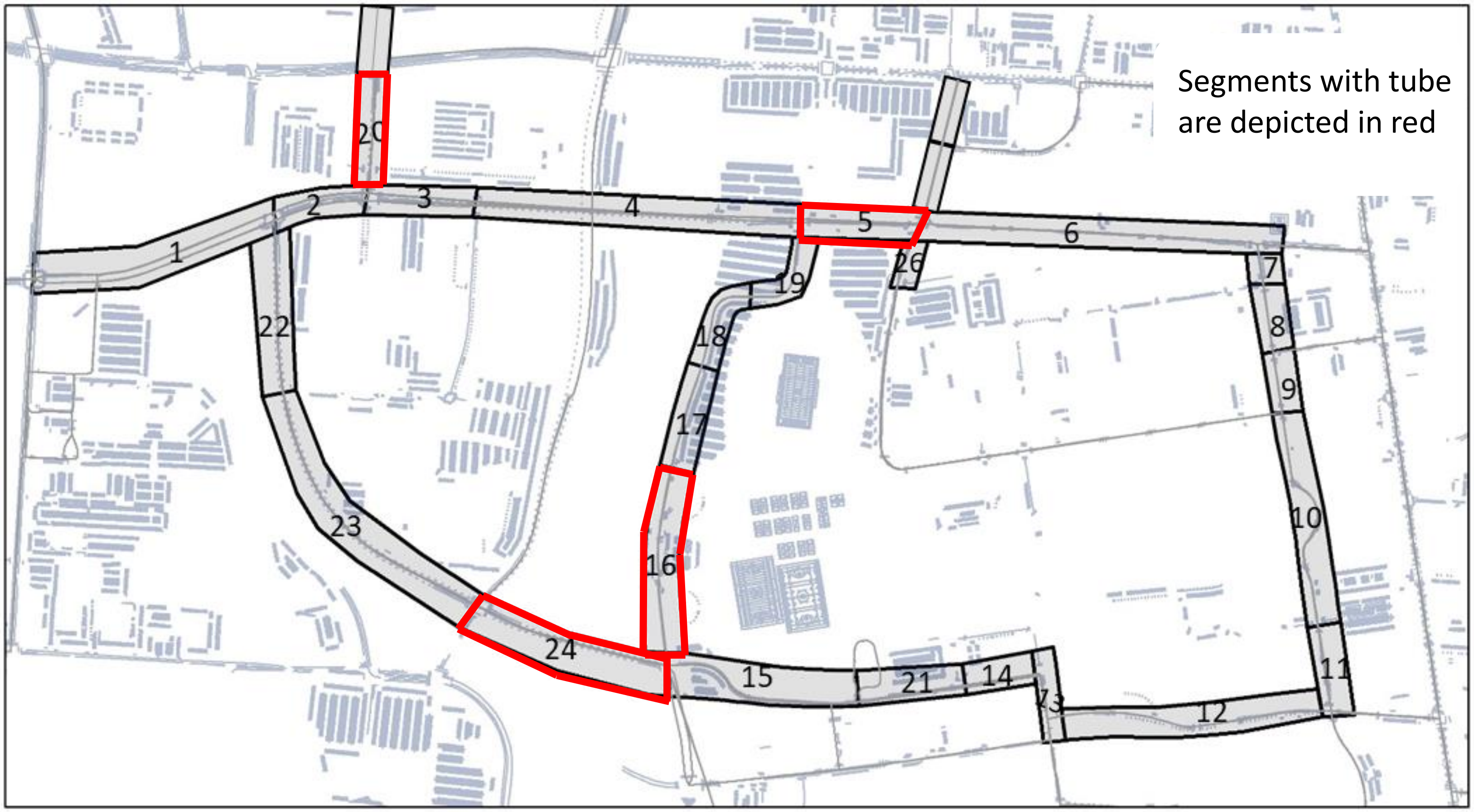

Figure 2.2: OSU campus roadway network used in 2019 data collection: red segments are those where road tubes were placed

2.2. Data Used in Empirical Study

In this study, traffic volumes are estimated from vehicle observations obtained from two types of data: video data (which will be denoted by *video*) and road tube data (which will be denoted by *tube*). These types of data are described in this section.

2.2.1. Video Data

The Ohio State University's (OSU) Transportation and Traffic Management (TTM) has installed several cameras on its buses for safety, security, and reliability purposes. The camera located on the rear, driver's side of the bus collected the imagery used in this study. This camera looks forward and can view vehicles to the left of the bus. As discussed below, vehicles travelling in the opposite direction of the bus are of interest in this study. The location of this rear, driver's side camera on one OSU Campus Area Bus Service (CABS) bus and a sample frame of the video imagery are shown in Figure 2.3.

Figure 2.3: Location on OSU CABS bus of camera (upper right) used to collect imagery that led to data used in this study; Circled vehicle in left image represents traffic of interest (direction opposite to bus travel)

In this thesis, processed video imagery obtained on October 25, 2018 and October 24, 2019 are used. Approximately 60 hours of video imagery obtained in 2018 and 72 hours of video imagery obtained in 2019 were previously processed to extract the vehicle count data of interest in this study. (More detail is provided below.) The video imagery used was collected between 7:00 a.m. and 7:00 p.m. on the two observation days. The times of bus traversals over the segment (what will be called "bus passes") in 2018 are presented in Figure.2.4. A dot (".") is indicated at the time that the bus first entered the segment. A similar plot for the 2019 data is presented in Figure 2.5.

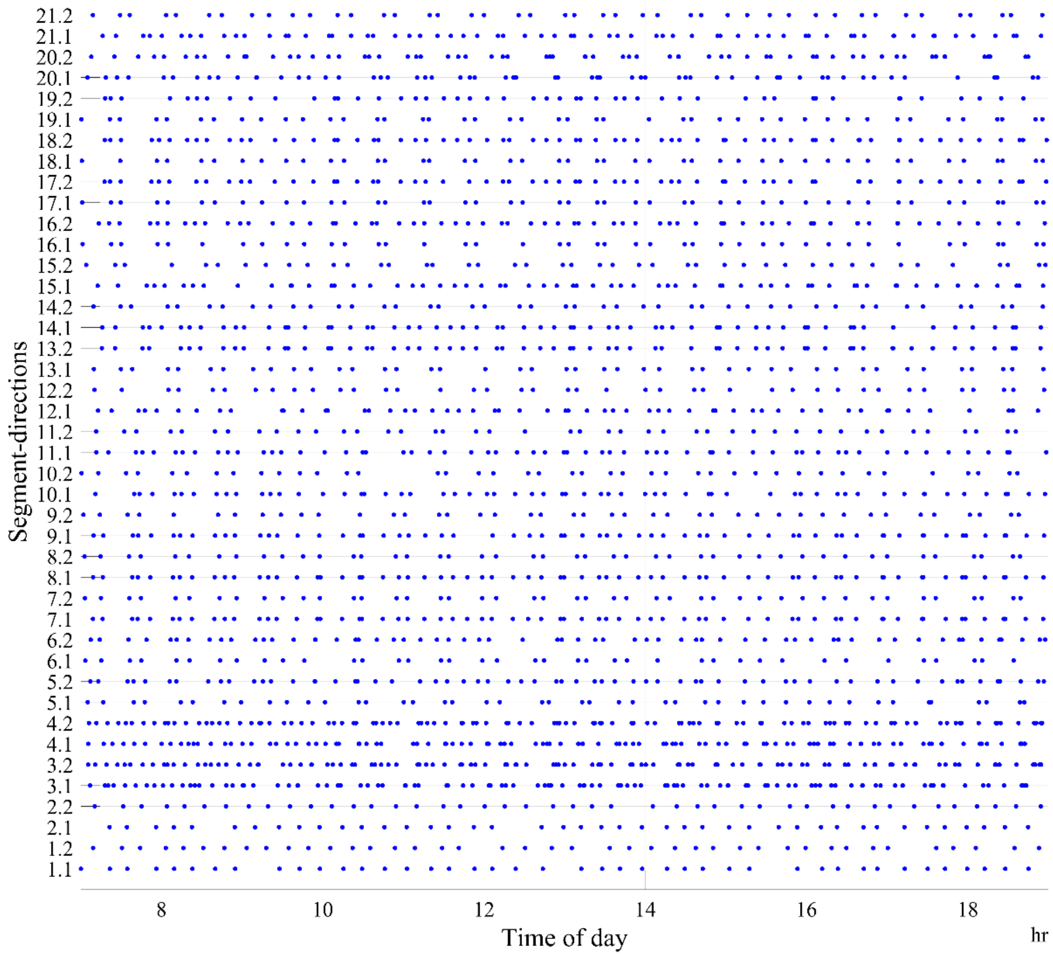

Figure 2.4: Indication of time-of-day, by segment, of bus passes collecting 2018 data

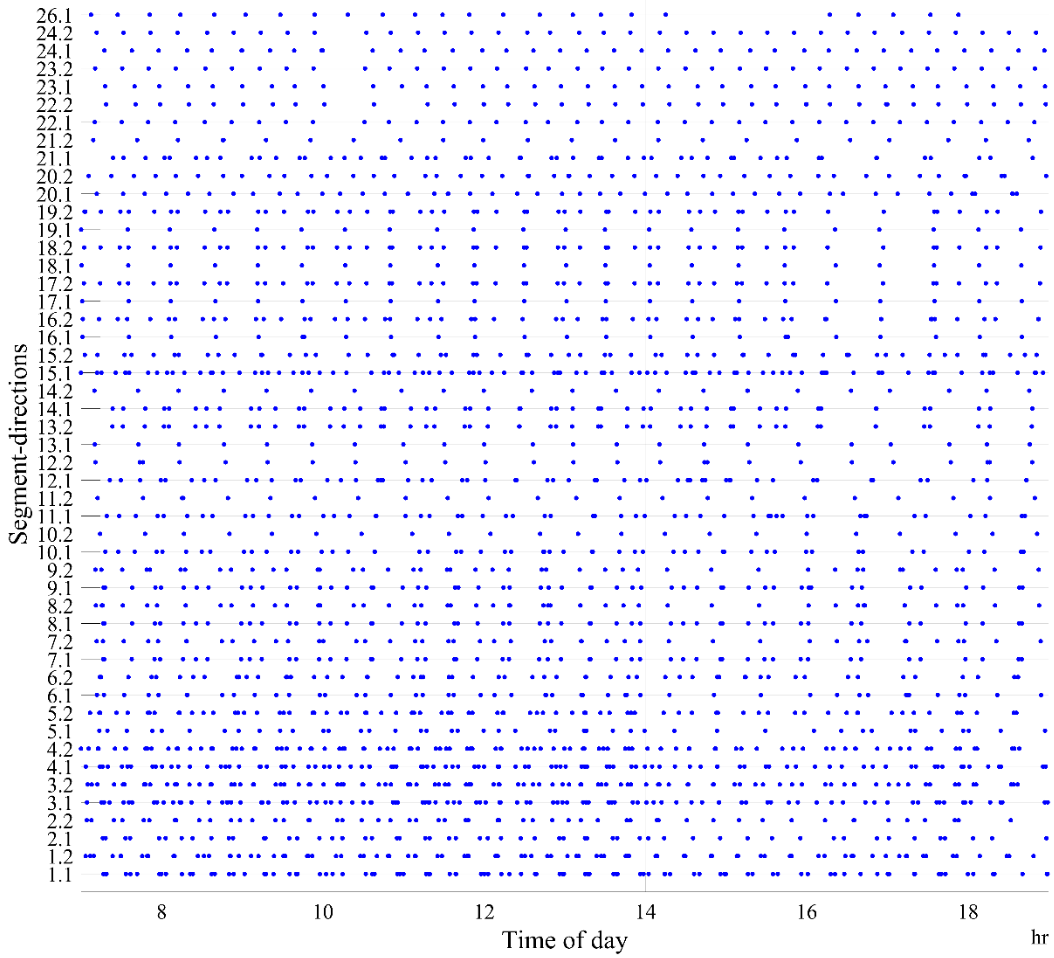

Figure 2.5: Indication of time-of-day, by segment, of bus passes collecting 2019 data

To convert the recorded video imagery into data that can be used to estimate traffic volumes, a Graphical User Interface (GUI) was previously developed by The Ohio State University Campus Transit Lab (CTL) researchers. This application provides an environment for an individual to watch the video and electronically record the frame number by "clicking" when a vehicle passes the bus in the opposite direction of its movement. Frame numbers corresponding to the beginning and end of the segment directions are similarly recorded using the GUI in what is termed "segmentation."

Comparing frame numbers of clicked vehicles to the frame numbers of the beginning and the end of the segments, which is done automatically, allows a determination of the number of vehicles n_i observed during bus pass i over the segment. Using the frame rate to convert the number of frames between the beginning and end of the segment (determined in the segmentation function) allows a determination of the time t_{1i} taken by the bus to traverse the segment in the direction opposite of the direction of traffic being observed on bus pass i. These values are used to estimate traffic volumes as described in Section 2.3. More details of the GUI applications and its steps can be found in McCord. et al. (2020). CTL graduate and undergraduate students and students from 2018 and 2019 offerings of the mixed graduate and undergraduate course, *CIVILEN 5720: Transportation Engineering Data Collection Studies*, used the GUI to process the video imagery used in this study.

2.2.2. Road Tube Data

The Mid-Ohio Regional Planning Commission (MORPC) laid road tubes on different subsets of the 2018 and 2019 network segments. The segments on which MORPC placed road tubes are listed in Table 2.1. The road tubes collected data that were provided in 15-minute intervals on the quarter-of-an-hour over a 24-hour period on the two days of data collection used in this study. These intervals will determine the 15- and 60-minute intervals used in the empirical studies in subsequent chapters. For these intervals, the road tube counts provided are used as the volume estimates. In the empirical study, road tube data collected between 7:30 a.m. to 6:30 p.m. are used.

Table 2.1: Roadway segments with road tubes on 2018 and 2019 data collection days

Year	Segments	Segment Name	Direction
2018	1.1	Wdy/Kenny to Wdy/Herrick	Eastbound
	1.2	Wdy/Herrick to Wdy/Kenny	Westbound
	4.1	Wdy/Coffey to Wdy/Cannon	Eastbound
	4.2	Wdy/Cannon to Wdy/Coffey	Westbound
	10.1	Hagerty/College to AnnieJohn/College	Northbound
	10.2	AnnieJohn/College to Hagerty/College	Southbound
	15.1	Herrick/Cannon to HUBdepart	Eastbound
	15.2	Herrick/HUBarrive to Herrick/Cannon	Westbound
	19.1	ShoePark3 to Wdy/Cannon	Northbound
	19.2	Wdy/Cannon to ShoePark3	Southbound
2019	5.1	Woody/Cannon to Woody/Tuttle	Eastbound
	5.2	Woody/Tuttle to Woody/Cannon	Westbound
	16.1	Cannon/JohnHerrick to ShoePark1	Northbound
	16.2	ShoePark1 to Cannon/JohnHerrick	Southbound
	20.1	Woody/Fyffe to Lane/Fyffe	Northbound
	20.2	Lane/Fyffe to Woody/Fyffe	Southbound
	24.1	Olentangy/JohnHerrick to Cannon/JohnHerrick	Eastbound
	24.2	Cannon/JohnHerrick to Olentangy/JohnHerrick	Westbound

2.3. Volume Estimation from Video on a Single Bus Pass

For each segment $S.s$ for which video imagery was processed, the output of the GUI application provides the number of vehicles n_i detected on $S.s$ for bus *pass* i and the time t_{1i} the bus took on pass i to traverse the segment in direction s' opposite direction s. A method described in McCord. et al. (2020) is used to estimate traffic volumes on $S.s$ from these observations. This approach considers the time t_{2i} that a "virtual vehicle" would take to traverse the segment S in direction s (the direction of the traffic being observed, which is opposite the direction of the bus). Various methods could be used to determine t_{2i} (see, e.g., McCord et al., 2020). The volume estimates used in this thesis are determined by calculating t_{2i} as the distance of the segment $S.s$ divided by the speed limit on that segment. Note that, with this approach, the value of t_2 does not depend on the bus pass. The subscript denoting the bus pass is used, however, to allow consideration of a more general case, where the time may depend on the bus pass. With the values of n_i, t_{1i}, and t_{2i}, the volume estimated from data on bus pass i during a time period T is given by:

$$Vol_i^T = \frac{n_i}{t_{1i} + t_{2i}} \times T \tag{2.3.1}$$

Where, T, t_{1i}, and t_{2i} are all in the same units of time. This thesis will consider times to be given in minutes and determine 15-minute (T = 15) and hourly (T = 60) traffic volumes.

Chapter 3: Empirical Evaluation Design and Aggregation by Arithmetic Averaging of Single Bus Pass Volumes in Discrete Time Intervals

As explained in Section 2.3, video-based volume estimates from a bus pass are derived from the number of vehicles observed during very short observation periods. Since it is likely to have multiple bus passes on a segment in a time period during which volume estimates are desired, there will be multiple volume estimates (one from each of the multiple bus passes) during this estimation period. The issue addressed in this thesis is the aggregation of these individual bus pass-derived volumes into an estimate for the estimated period volume. In this chapter, averaging the volumes estimated from the individual bus passes by discrete time intervals is considered. First, the design characteristics of the empirical evaluations of all aggregation methods considered in this thesis are presented in Section 3.1. Then, in Section 3.2.1, what will be referred to as Method 1 is presented, along with empirical results. This method consists of taking averages of the bus pass-derived volumes in the interval of interest and expands them to the interval duration. What will be referred to as Method 2 is presented in Section 3.2.2. This method transforms each bus pass-derived volume through a regression model before aggregating the set of transformed volumes by averaging.

3.1. Evaluation Metrics, Estimation Periods, and Cases

3.1.1. Evaluation Metrics

In this thesis, estimates derived from different aggregation methods will be evaluated against road tube volumes for 15- and 60-minute time periods. To quantify the accuracy of an estimate, three metrics based on differences between the video-derived volumes for an estimation period and the road tube volume for the same estimation period are used: *difference*, *absolute difference*, and *absolute relative error*.

The *difference* metric $Diff_{t_{beg}}^{t_d}$ represents the magnitude and direction of the difference between the video-based estimate and the corresponding road tube volume for an estimation period beginning at time t_{beg} (denoted in HH:MM) and having duration t_d (denoted in minutes):

$$Diff_{t_{beg}}^{t_d} = V_{t_d,t_{beg}}^{Video} - V_{t_d,t_{beg}}^{Tube} \tag{3.1.1}$$

where $V_{t_d,t_{beg}}^{Video}$ is the estimated volume from the video data for the indicated estimation period after aggregating the data from the individual bus passes, and $V_{t_d,t_{beg}}^{Tube}$is the volume obtained from the road tube data for the same estimation period. Summarizing *difference* metric values is useful to assess whether the video-based estimates tend to over- (positive difference) or underestimate (negative difference) the corresponding tube-based volumes serving as the reference.

The *absolute difference* metric $Abs\ Diff_{t_{beg}}^{t_d}$ is the absolute value of $Diff_{t_{beg}}^{t_d}$:

$$Abs\ Diff_{t_{beg}}^{t_d} = Abs\left(V_{t_d,t_{beg}}^{Video} - V_{t_d,t_{beg}}^{Tube}\right) \tag{3.1.2}$$

where the notation is the same as used above and *Abs*(.) represents the absolute value of the argument. *Absolute difference* represents the magnitude of the "error" as measured by the difference between the estimated video-based volume and the corresponding road tube volume. Summarizing the *absolute difference* values across the estimation periods and segments emphasizes the size of the errors, since positive and negative values will not cancel out as they do when summarizing *difference* values.

The third metric is *absolute relative error*. This metric is used to measure errors on a similar scale across different magnitudes of volume. The absolute relative error for an estimation period beginning at time t_{beg} (denoted in HH:MM) and having duration t_d (denoted in minutes) is defined as:

$$ARE_{t_{beg}}^{t_d} = Abs\left[\frac{\left(V_{t_d,t_{beg}}^{Video} - V_{t_d,t_{beg}}^{Tube}\right)}{V_{t_d,t_{beg}}^{Tube}}\right] \quad (3.1.3)$$

where the same notation is again used.

3.1.2 Estimation Periods Considered

These metrics are computed for each of the ten and eight segments in 2018 and 2019, respectively on which road tubes and video-based volumes were previously estimated, and for the estimation periods of t_{beg} = 7:30,…, 18:15 for $t_d = 15$ (minutes) and t_{beg} = 7:30 , 8:30 ,…, 17:30, for $t_d = 60$ (minutes). It should be noted that, although video data were collected beginning at 7:00 and ending at 19:00, metrics are only considered for periods beginning at 7:30 and ending at 18:15 for $t_d = 15$ (minutes) and beginning at 7:30 and ending at 17:30 for $t_d = 60$ (minutes). This specification will allow the video data to be used to interpolate across the beginnings and ends of periods when using methods considered in Chapter 4.

Some intervals $[t_{beg}, t_{beg} \ + 15]$ contained in the periods of interest have no bus passes on some segments, and metrics could, therefore, not be computed for the methods presented in Chapter 3 (Methods 1 and 2). To allow comparisons among the different aggregation methods, these periods will not be considered when summarizing metric values for any of the methods considered in this thesis. The start time of the 15-minute periods with no bus passes on a segment are provided in Table 3.1. All the 60-minute periods have at least one bus pass; therefore, all 60-minutee periods will be considered when summarizing metric values.

Table 3.1: Start time t_{beg} of 15-minute periods containing no bus passes, and total number of intervals containing at least one bus pass, by segment

Data set	Segments	Start time of periods containing no bus passes - t_{beg}	Total number of 15-minute periods with at least one bus pass (maximum possible: 44 intervals)
2018	Seg 1.1	9:00, 10:30, 12:45, 14:00, 14:45, 15:30	38
	Seg 1.2	7:45, 11:45, 12:30, 13:15, 15:30, 16:00, 16:30, 17:15, 18:15	35
	Seg 4.1	10:45	43
	Seg 4.2	None	44
	Seg 10.1	9:45, 10:30, 14:30, 15:15	40
	Seg 10.2	7:45, 10:00, 10:30, 10:45, 11:00, 12:15, 12:45, 13:15, 13:45, 14:15, 16:30, 17:00, 17:15, 17:45, 18:15	29
	Seg 15.1	7:30, 9:00, 15:45, 16:45	40
	Seg 15.2	7:45, 8:15, 8:45, 11:00, 11:30, 12:00, 14:15, 15:45, 17:30, 18:00	34
	Seg 19.1	7:30, 8:45, 9:15, 9:45, 10:15, 10:45, 12:00, 12:30, 13:30, 13:45, 16:45, 18:00	32
	Seg 19.2	7:30, 7:45, 9:30, 12:15, 14:45, 15:45, 16:30, 16:46, 17:30	35
2019	Seg 5.1	8:30, 9:45, 12:30, 14:00, 14:30, 15:00, 15:30, 15:45, 17:00	35
	Seg 5.2	12:15, 16:45, 18:15	41
	Seg 16.1	7:45, 8:15, 8:45, 9:15, 10:00, 10:30, 11:00, 11:30, 12:00, 12:30, 12:45, 13:30, 13:45, 14:15, 14:45, 15:15, 16:00, 16:30, 17:00, 17:15, 17:45, 18:15	22
	Seg 16.2	9:30, 11:30, 12:30, 13:30, 16:15, 16:30, 17:00, 17:15	36
	Seg 20.1	10:30, 14:00, 16:00, 16:30, 17:15, 18:15	38
	Seg 20.2	9:00, 11:00, 12:15, 15:15, 16:45, 18:00	38
	Seg 24.1	8:00, 9:00, 10:00, 10:15, 11:15, 12:00, 13:00, 14:00 , 15:00, 16:00, 17:00, 18:00	32
	Seg 24.2	8:15, 9:15, 10:00, 10:15, 11:30, 12:15, 13:30, 14:30, 15:30, 16:30, 17:30	33

3.1.3 Cases to Handle Exceptionally High or Low Volumes from Individual Bus Passes

An estimated volume from a single bus pass may be considered to be exceptionally high or low. An exceptionally high volume would result when an excessive number of vehicles is observed during the short duration of the bus pass. From manual observation of the video data, this situation tends to occur when there is a queue on the roadway segment in the opposite direction of the bus movement (e.g., at intersections when the traffic light is red). In this thesis, exceptionally high volumes are considered to be volumes greater than 10 vehicles per lane per minute. This value of 10 vehicles per lane per minute is based on observation of road tube data and standard values presented in the Highway Capacity Manual (National Research Council, 2000), the discussion of which is outside the scope of this thesis. An exceptionally low volume would result when there are no, or very few, vehicles on the roadway segment during the short time of the bus pass. It would be unreasonable to believe that no vehicles, for example, observed in a very short time (t_1 minutes defined in Section 2.2.1) should lead to an estimate of zero volume in a longer period of t_d duration. In this thesis, exceptionally low volumes are considered to occur when no vehicles are observed on the bus pass.

Three "Cases" are considered for handling these exceptionally high and low volumes obtained from the single bus pass:

- Case 1: Retain the exceptionally low and high bus pass volumes as estimated from Equation (2.3.1) when applying an aggregation method
- Case 2: Remove exceptionally low (i.e., 0) or exceptionally high (i.e., greater than 10 vehicles/lane/minute) bus pass volumes before applying aggregation

- Case 3: Set exceptionally low (i.e., 0) bus pass volumes to 0.5 vehicles/lane/minute and exceptionally high (i.e., greater than 10 vehicles/lane/minute) bus pass volumes to 10 vehicles/lane/minute before applying aggregation

Exploring other definitions of exceptionally high and low bus pass volumes and for modifying these volumes before aggregation could be a topic for future research.

3.2 Arithmetic Averaging of Volume Estimates from Individual Bus Passes

3.2.1 Method 1: Arithmetic Averaging of Volume Estimates Directly Obtained from Bus Passes

Perhaps the most straightforward method to aggregate volumes from the individual bus passes into an estimated volume for a given estimation period is to take the simple (arithmetic) average of the volumes derived from all the individual bus passes in the interval corresponding to the estimation period and expand to the interval duration. Taking the arithmetic average of volume estimates directly obtained from individual bus passes is the method used in McCord et al. (2020) and will be referred to as Method 1 in this thesis. Summary statistics of the metrics obtained when using Method 1 are presented in Table 3.2. for estimation of 15-minute volumes. The table separates the statistics by Case (for treating exceptionally high and low bus pass volume estimate). For example, in the row titled Case 1, 2018 pooled, the table indicates that the summary statistic of the metrics pooled across segments and 15-minute periods in the 2018 data under Case 1 are based on 370 observations (segment-15-minute periods). This row presents the mean (average), standard deviation, and median, respectively, of the 370 *difference* values as 4.423, 26.439, and 1.435 vehicles, of the 370 *absolute difference* values as 18.234, 19.628, and 12.135

vehicles, and of the 370 *absolute relative error* values as 0.324, 0.355, and 0.236. The second row presents statistics obtained under Case 1 when using the 2019 data, which contains 275 segment-15-minute periods. The third row presents the statistics obtained when pooling all the 2018 and 2019 metrics. The pooled set of data contains 645 (= 370 + 275) segment-15-minute periods. Then, the table presents statistics obtained under Cases 2 and 3 in a similar fashion. It is noted that the number of segment-15-minute periods is the same in Cases 1 and 3 (where all passes with exceptionally low or high values are included in the analysis, but after modification in Case 3). However, removing the bus passes with exceptionally low or high volumes in Case 2 leads to no bus passes remaining in some periods, which are then not considered for analysis. Therefore, Case 2 contains fewer segment-15-minute periods than Cases 1 and 3. Statistics for 60-minute estimation periods are presented in Table 3.3 in the same format. The elimination of bus passes with exceptionally low or high volumes does not lead to any 60-minute period having no bus passes. Therefore, the total numbers of intervals considered under Case 2 is the same as in Cases 1 and 3. Summary statistics by segment are presented in Appendix A.

Table 3.2: Summary statistics of metrics obtained with Method 1 (arithmetic average of volume estimates from bus passes), by Case and Data set, for 15-minute periods

Case Number	**Data set**	**Number of Segment Direction**	**Difference**			**ABS Difference**			**ARE**		
			Mean	**Standard Deviation**	**Median**	**Mean**	**Standard Deviation**	**Median**	**Mean**	**Standard Deviation**	**Median**
1	2018 Pooled	370 Segment-15-minute periods	4.423	26.439	1.435	18.234	19.628	12.135	0.324	0.355	0.236
	2019 Pooled	275 Segment-15-minute periods	8.953	31.785	4.464	21.721	24.843	15.214	0.398	0.375	0.331
	2018 and 2019 Pooled	645 Segment-15-minute periods	6.332	28.876	2.962	19.704	22.027	13.149	0.355	0.365	0.268
2	2018 Pooled	347 Segment-15-minute periods	1.827	20.568	1.066	15.031	14.136	10.710	0.291	0.330	0.217
	2019 Pooled	268 Segment-15-minute periods	5.608	23.684	4.252	18.438	15.8502	14.596	0.359	0.297	0.323
	2018 and 2019 Pooled	615 Segment-15-minute periods	3.449	22.020	2.142	16.493	14.978	12.199	0.320	0.318	0.256
3	2018 Pooled	370 Segment-15-minute periods	2.507	22.088	1.233	16.086	15.321	11.937	0.304	0.338	0.221
	2019 Pooled	275 Segment-15-minute periods	7.248	25.609	4.464	19.860	17.682	15.047	0.373	0.305	0.327
	2018 and 2019 pooled	645 Segment-15-minute periods	4.505	23.733	2.906	17.677	16.451	12.916	0.333	0.326	0.260

Table 3.3: Summary statistics of metrics obtained with Method 1 (arithmetic average of volume estimates from bus passes), by Case and Data set, for 60-minute periods

Case Number	Data set	Number of Segment Direction	Difference			ABS Difference			ARE		
			Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	2018 Pooled	110 Segment-60-minute periods	19.755	74.731	11.937	51.49	57.472	32.733	0.226	0.238	0.154
	2019 Pooled	88 Segment-60-minute periods	36.084	108.969	16.503	70.922	90.020	46.463	0.301	0.308	0.232
	2018 and 2019 Pooled	198 Segment-60-minute periods	27.013	91.653	13.638	60.126	74.162	36.240	0.259	0.273	0.188
2	2018 Pooled	110 Segment-60-minute periods	9.853	55.965	9.093	39.665	40.522	27.727	0.204	0.266	0.136
	2019 Pooled	88 Segment-60-minute periods	18.849	71.195	13.089	54.452	49.281	40.460	0.242	0.190	0.211
	2018 and 2019 Pooled	198 Segment-60-minute periods	13.867	63.202	11.6/16	46.262	45.128	33.418	0.221	0.235	0.154
3	2018 Pooled	110 Segment-60-minute periods	11.793	57.423	11.937	42.518	40.166	32.239	0.205	0.228	0.149
	2019 Pooled	88 Segment-60-minute periods	28.061	83.039	13.638	62.047	61.627	42.959	0.270	0.222	0.214
	2018 and 2019 Pooled	198 Segment-60-minute periods	19.023	70.252	12.683	51.198	51.619	35.174	0.234	0.227	0.170

Looking at the summary statistics of metrics by segment in Appendix A, one sees that there are differences in the metric statistics depending on the segment. Of particular note, some segments have positive mean and median differences, meaning that the video-based volumes are on average (mean) or in greater proportion (median) larger than the road tube-based volumes, while other segments have negative mean and median differences, meaning that video-based volumes are on average or in greater proportion lower than the road tube-based volumes. This occurs for both 15- and 60-minute estimates and in each of Cases 1-3. From Tables 3.2 and 3.3, one sees that when pooling across metrics obtained for the 2018 segment-periods, pooling across metrics obtained for the 2019 segment-periods, and pooling across all segment-periods for all segment-periods (i.e., combining the 2018 and 2019 data sets), the mean and median differences are positive for all the cases. Since the averages (means) are relatively small compared to the standard deviations, the positive differences are not considered striking.

The magnitude of the mean and median of *ARE*'s and *absolute differences* of all the 15- and 60-minute estimation periods are generally higher in the pooled 2019 data set than in the pooled 2018 data set in all three cases. The lower errors when pooling segments contained in the 2018 data set compared to pooling segments in the 2019 data set may result from a larger number of bus passes available for estimation in 2018. There are 597 passes in 2018, which leads to an average of 59.7 passes across the 10 segments in the 2018 data set. In 2019, there are 371 passes, which leads to an average of 46.4 passes across the 8 segments in the 2019 data set. Since different segments are considered in the two data sets, segment-specific characteristics could also be affecting the quality of the video-based volumes. Analytically investigating the quality of the estimates as a function of number of bus passes and segment-direction could be a topic for future research.

This thesis is not exploring the benefits of different ways to handle exceptionally high or low volumes. However, considering the summary statistics in Tables 3.2 and 3.3 by Case indicates that Case 2 is better than Case 3, which is better than Case 1 for each dataset individually and for pooled datasets. For example, the mean *ARE's* in 15-minute periods in 2018 for Case 2, Case 3, and Case 1, respectively are 0.291, 0.304, and 0.324. Lower errors in Case 2 might be because of removing exceptionally low and high volumes, which would cause large errors, from the analysis. In addition, setting the unreasonably low and high volumes to 0.5 and 10 vehicle per lane per minute would tend to reduce the error associated with the original values and make Case 3 better than Case 1.

Comparing the mean and median of *ARE's* for the corresponding data sets in Tables 3.2 and 3.3 reveals that the 60-minute estimation periods result in lower errors than the 15-minute estimation periods, as expected. That is, having more bus pass volumes in 60-minute interval provides more accurate aggregated estimates, all other things being equal. The mean and median values when considering *absolute difference* is larger for the 60-minute estimations than for the 15-minute estimations, however. This is expected, since more vehicles would be on a segment in a 60-minute period than in a 15-minute period, so the magnitude of the estimate (the volume) would be larger in 60 minutes than in 15 minutes.

3.2.2. Method 2: Arithmetic Averaging of Volume Estimates from Bus Passes after Regression-based Transformation

A second method is presented that uses arithmetic averaging to aggregate volumes obtained from individual bus passes. In this method, which will be referred to as Method 2, each volume estimated from video on an individual bus pass using Equation (2.3.1) is transformed through a regression model before aggregating with the arithmetic average. Regression-based transformations are considered for other purposes outside of the scope of this thesis. For this thesis the transformations are primarily used to provide a second set of data to be analyzed. Different specifications of the regression model are considered in Section 3.2.2.1, and applications of the chosen specifications with the aggregation results for 15-minute and 60-minute estimation periods are presented in Section 3.2.2.2.

3.2.2.1. Regression specifications

Different specifications of a model that maps a 15-minute video-based volume estimate from a single pass into a 15-minute road tube volume for the 15-minute period that contains the time of the pass are considered and evaluated using the pooled 2018 and 2019. Based on the results of the different specifications, two specifications are choses for use with Method 2.

The first specification (denoted as SPEC 1) is a simple linear specification of the following form:

$$True\ Volume_i = \beta_0 + \beta_1 \times Video\ Bus\ Pass\ Volumes_i \tag{3.2.1}$$

where the $True\ Volume_i$ corresponds to what is believed to be the true volume in the 15-minute period $[t_{beg}, t_{beg} + 15]$ during which the bus pass i occurred, and the tube volume during this interval is used as a substitute of the true volume. $Video\ Bus\ Pass\ Volume_i$ is the $Vol\ _i^{15}$ estimated from the video data on bus pass i using Equation (2.3.1) with $T\ =\ 15.$

Specifications 2 through 20 are of the following form:

$$\begin{aligned} True\ Volume_i \\ &= \beta_0 + \beta_1 \times Video\ Bus\ Pass\ Volume_i \\ &+ \beta_2 \times Video\ Bus\ Pass\ Volume_i^{\alpha} \end{aligned} \qquad (3.2.2 – 3.2.20)$$

where $True\ Volume_i$ and $Video\ Bus\ Pass\ Volume_i$ are the same values used in SPEC 1, and α is an exponent that is pre-set to allow estimation of parameters $\beta_0,\ \beta_1$, and β_2. SPEC 2 – SPEC 20 consider α values 0.1, 0.2, …, 1.9, 2.0, respectively, but not α of 1 (since $\alpha\ = 1$ would lead to a linear specification like SPEC 1).

In this section, SPEC 1 and one of SPEC 2-20 are considered for use in Method 2. To determine which of SPEC 2-20 is chosen for the analysis, each is estimated separately for the three Cases defined in Section 3.1.3 using the pooled 2018 and 2019 data. For each specification, the estimated coefficients, p-values, and adjusted R-squared are tabulated in Tables A.7-A.9 of Appendix A. In addition to these traditionally reported values, the *Tube Volumes* using the estimated coefficients and *Video Bus Pass Volumes* of 7.5 and 150 vehicles/ln/15-minutes are calculated and tabulated, where it is desired that calculated values at *Video Bus Pass Volume* = 7.5 and at *Video Bus Pass Volume* = 150 are closer to, rather than farther from, low and high volumes default values of 7.5 and 150, respectively.

Looking at the regression results obtained from different specifications in Appendix A, one sees that adjusted R-squared values are very similar for all specifications. The levels of significance of the estimated coefficients also do not vary much across the specifications. Also, when using *Video Volumes* at 7.5 and 150 vehicles/ln/15-minutes, SPEC 20 generally calculates *Tube Volumes* closest to the input values. Therefore, with no clear “best” specification emerging, SPEC 20 (with α of 2.0) is chosen from among all the SPEC 2 – SPEC 20. More detailed analysis of regression model specification could be a topic for future research. The estimation and calculation results for SPEC 1 and SPEC 20 are presented in Table 3.4 to illustrate the statistics and calculated values considered when comparing across SPEC 2-20. As will be seen in the next section, when investigating Method 2, SPEC 1 and SPEC 20 will be re-estimated so that the same data set is not used in both the estimation of the coefficients and the investigation of its performance.

Table 3.4: Estimated coefficients, t-statistics, p-values, adjusted R-squared values, and Tube Volumes calculated at Video Volumes of 7.5 and 150 vehicles/ln/15minutes obtained from SPEC 1 and SPEC 20 for Cases 1-3 using pooled 2018 and 2019 Data sets

Cases	**Specifications**	**Coefficients**		**t-statistics**	**p-value**	**Adjusted R-squared**	**Calculated Tube Volume using Video Volume of 7.5 veh/ln/15-min**	**Calculated Tube Volume using Video volume of 150 veh/ln/15-min**
Case 1	SPEC 1	β_0	31.399	31.74	2e-16	0.4786	34.375	90.919
		β_1	0.397	29.81	2e-16			
	SPEC 20	β_0	25.946	17.893	2e-16	0.4917	30.105	91.035
		β_1	0.561	16.099	2e-16			
		β_2	-0.0009	-5.088	4.35e-07			
Case 2	SPEC 1	β_0	27.804	24.19	2e-16	0.4284	31.274	97.213
		β_1	0.463	26.11	2e-16			
	SPEC 20	β_0	28.800	13.911	2e-16	0.4280	32.036	98.801
		β_1	0.424	6.331	3.84e-10			
		β_2	0.0003	0.60	0.549			
Case 3	SPEC 1	β_0	27.429	25.64	2e-16	0.4958	30.980	98.460
		β_1	0.474	30.85	2e-16			
	SPEC 20	β_0	29.960	16.241	2e-16	0.4968	32.864	100.982
		β_1	0.383	6.834	1.46e-11			
		β_2	0.0006	1.686	0.0922			

In SPEC 1 and SPEC 20, all the coefficients are significant except β_2 in SPEC 20 in Case 2. Case 2 also results in the lowest adjusted R-squared value. As discussed, in Case 2 the exceptionally high and low volumes are removed from analysis. Removing these exceptionally high and low values may make the higher order term ($Video\ Bus\ Pass\ Volume_i^2$) of less benefit than when these very high and low values are included, and therefore may lead to the insignificant value of β_2. For the same reason, the adjusted R-squared value may decrease when the independent variable has less effect on the dependent variable. More detailed interpretation of the regression results could be a topic for future research.

3.2.2.2. Application of Method 2

This section presents the empirical results obtained from Method 2, that is, aggregating individual bus pass volumes using arithmetic average after being transformed by SPEC 1 or by SPEC 20. Different data sets are used to estimate coefficients of the regression (β_0 and β_1 for SPEC 1, and β_0, β_1, and β_2 for SPEC 20) by Case and to evaluate the performance when using Method 2. Specifically, the SPEC 1 and SPEC 20 coefficients are estimated using the video volumes obtained from individual bus passes and the corresponding road tube volumes from the 2019 data set, and the estimated coefficients are used to transform the bus pass estimated volumes from the 2018 data set. Then, these 2018 transformed volumes are arithmetically averaged in the same 15-and 60-minute intervals used in Section 3.1.2 and compared to the corresponding road tube volumes to obtain *difference, absolute difference,* and *absolute relative difference* metrics as was done in Section 3.2.1. The process is then repeated by using 2018 data to estimate regression coefficients, using these 2018 estimated coefficients to transform the bus pass estimated volumes from the 2019 data set, arithmetically averaging these 2019 transformed bus pass volumes in the same 15- and 60-minute intervals, and calculating the metrics based on these aggregated 2019 transformed volumes. Table 3.5 presents the summary regression results from the different estimation data sets obtained from SPEC 1 and SPEC 20 in Cases 1-3.

Table 3.5: Estimated coefficients, t-statistics, p-values, and adjusted R-squared values obtained from SPEC 1 and SPEC 20 for Cases 1-3 and each Data set

Cases	Estimation Data set	Specifications	Coefficients		t-statistics	p-value	Adjusted R-squared
Case 1	2018	SPEC 1	β_0	26.963	21.57	2e-16	0.5766
			β_1	0.490	28.50	2e-16	
		SPEC 20	β_0	26.081	13.427	2e-16	0.5761
			β_1	0.517	10.704	2e-16	
			β_2	-0.0001	-0.593	0.553	
	2019	SPEC 1	β_0	36.913	25.02	2e-16	0.3595
			β_1	0.277	14.45	2e-16	
		SPEC 20	β_0	28.347	14.189	2e-16	0.4158
			β_1	0.533	11.561	2e-16	
			β_2	-0.001	-6.048	3.61e-09	
Case 2	2018	SPEC 1	β_0	25.361	16.87	2e-16	0.4632
			β_1	0.519	22.02	2e-16	
		SPEC 20	β_0	28.120	9.887	2e-16	0.4635
			β_1	0.418	4.561	6.25e-06	
			β_2	0.0007	1.143	0.254	
	2019	SPEC 1	β_0	30.867	17.64	2e-16	0.3865
			β_1	0.390	14.80	2e-16	
		SPEC 20	β_0	30.037	10.145	2e-16	0.3849
			β_1	0.422	4.379	10.59e-05	
			β_2	-0.0002	-0.347	0.728	

Continued

Table 3.5 Continued

Cases	Estimation Data set	Specifications	Coefficients		t-statistics	p-value	Adjusted R-squared
Case 3	2018	SPEC 1	β_0	23.541	16.87	2e-16	0.5592
			β_1	0.560	27.51	2e-16	
		SPEC 20	β_0	30.140	12.159	2e-16	0.5660
			β_1	0.328	4.377	1.42e-05	
			β_2	0.002	3.211	0.0014	
	2019	SPEC 1	β_0	32.548	20.99	2e-16	0.4219
			β_1	0.357	16.46	2e-16	
		SPEC 20	β_0	29.915	11.785	2e-16	0.4230
			β_1	0.455	5.860	1.03e-08	
			β_2	-0.0007	-1.309	0.191	

From Table 3.5, it is seen that the values of the estimated coefficients depend on the different data sets used. What will be of interest is how the estimated models behave when applied to a different data set. Similar to what was seen in the regression summary when using pooled data sets, the adjusted R-squared value is generally the lowest value for the same specification in Case 2, except in SPEC 1 in Case 1. The coefficients are generally significant, except the β_2 coefficient which is insignificant in a few estimations. Specifically, this coefficient is insignificant in SPEC 20 in Cases 1 and 2 when using 2018 dataset, and in SPEC 20 in Cases 2 and 3 when using 2019 dataset.

Developing good regression models is not the focus of this thesis. The focus is to evaluate different approaches to aggregate a common set of bus pass volumes, in this case a set of bus pass volumes transformed by a regression model. Summary statistics of the metrics obtained using Method 2 with SPEC 1 and SPEC 20 for 15-minute estimation periods are calculated and presented in Tables 3.6 and 3.7, respectively. Similar tables for 60-minute estimation period are provided as Tables 3.8 and 3.9 (It is noted that the same coefficients estimated using 15-minute tube volumes, are used in the transformations that are aggregated over 60-minute periods, the results of which are presented in Tables 3.8-3.9) Similar to the tables in Section 3.2.1, these tables separate the statistics by Case to handle exceptionally high or exceptionally low bus pass volumes. (Elimination or transforming of the bus pass volumes was conducted before estimating the regression models.) In the row titled Est 2019-Tran 2018, the table presents the summary statistics of metrics obtained when transforming 2018 data set volumes through the regression model estimated using the 2019 data set, and in the row titled Est 2018-Tran 2019, the table presents the summary statistics of metrics obtained when transforming 2019 data set volumes through the regression model estimated using the 2018 data set. Summary statistics of the pooled set of Est 2019-Tran 2018 and Est 2018-Tran 2019 metrics are also presented in the last row of each Case. Summary statistics of the metrics for each segment are provided in Appendix A.

Table 3.6: Summary statistics of metrics obtained with Method 2 (arithmetic average of regression-based transformations of individual bus pass volumes) using SPEC 1 for 15-minute period estimates, by Case

Case Number	Estimation-Transformation	Number of Segment Direction Periods	Difference			ABS Difference			ARE		
			Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	Est 2019-Tran 2018	370 Segment-15-minute periods	-3.562	18.430	0.196	14.559	11.826	11.989	0.269	0.220	0.229
	Est 2018-Tran 2019	275 Segment-15-minute periods	3.255	17.459	3.148	12.990	12.087	9.818	0.266	0.265	0.189
	Pooled (Est 2019- Tran 2018 and Est 2018-Tran 2019)	645 Segment-15-minute periods	-0.689	18.326	1.885	13.898	11.952	11.264	0.268	0.240	0.210
2	Est 2019-Tran 2018	347 Segment-15-minute periods	-2.144	16.458	0.500	12.864	10.465	10.474	0.252	0.224	0.212
	Est 2018-Tran 2019	268 Segment-15-minute periods	2.030	15.916	2.670	12.290	10.2874	10.251	0.260	0.252	0.193
	Pooled (Est 2019- Tran 2018 and Est 2018-Tran 2019)	615 Segment-15-minute periods	-0.353	16.346	1.583	12.618	10.385	10.397	0.255	0.237	0.202
3	Est 2019-Tran 2018	370 Segment-15-minute periods	-3.625	18.360	-0.226	14.104	12.281	11.086	0.257	0.218	0.218
	Est 2018-Tran 2019	275 Segment-15-minute periods	3.312	16.148	3.260	12.710	10.470	10.568	0.261	0.247	0.200
	Pooled (Est 2019-Tran 2018 and Est 2018-Tran 2019)	645 Segment-15-minute periods	-0.701	17.783	1.699	13.517	11.564	10.847	0.259	0.231	0.209

Table 3.7: Summary statistics of metrics obtained with Method 2 (arithmetic average of regression-based transformations of individual bus pass volumes) using SPEC 20 for 15-minute period estimates, by Case

Case Number	Estimation-Transformation	Number of Segment Direction Periods	Difference			ABS Difference			ARE		
			Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	Est 2019-Tran 2018	370 Segment-15-minute periods	-3.424	18.761	0.415	14.293	12.604	11.367	0.261	0.222	0.223
	Est 2018-Tran 2019	275 Segment-15-minute periods	3.260	17.335	3.147	12.971	11.929	10.150	0.265	0.262	0.191
	Pooled (Est 2019- Tran 2018 and Est 2018-Tran 2019)	645 Segment-15-minute periods	-0.607	18.458	1.719	13.736	12.332	10.874	0.263	0.239	0.203
2	Est 2019-Tran 2018	347 Segment-15-minute periods	-2.111	16.479	0.614	12.868	10.486	10.434	0.252	0.225	0.212
	Est 2018-Tran 2019	268 Segment-15-minute periods	2.103	15.900	2.642	12.317	10.2461	9.900	0.262	0.259	0.195
	Pooled (Est 2019- Tran 2018 and Est 2018-Tran 2019)	615 Segment-15-minute periods	-0.303	16.354	1.600	12.632	10.379	10.287	0.256	0.240	0.201
3	Est 2019-Tran 2018	370 Segment-15-minute periods	-3.553	18.561	0.220	14.136	12.523	10.972	0.257	0.220	0.218
	Est 2018-Tran 2019	275 Segment-15-minute periods	3.330	16.342	2.977	12.789	10.678	10.052	0.265	0.263	0.192
	Pooled (Est 2019- Tran 2018 and Est 2018-Tran 2019)	645 Segment-15-minute periods	-0.652	17.971	1.796	13.568	11.791	10.662	0.260	0.239	0.206

Table 3.8: Summary statistics of metrics obtained with Method 2 (arithmetic average of regression-based transformations of individual bus pass volumes) using SPEC 1 for 60-minute period estimates, by Case

Case Number	Estimation-Transformation	Number of Segment Direction Periods	Difference			ABS Difference			ARE		
			Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	Est 2019-Tran 2018	110 Segment-60-minute periods	-14.187	67.495	2.371	53.096	43.746	39.784	0.234	0.175	0.210
	Est 2018-Tran 2019	88 Segment-60-minute periods	10.091	59.654	12.610	44.042	41.227	35.763	0.209	0.210	0.175
	Pooled (Est 2019- Tran 2018 and Est 2018-Tran 2019)	198 Segment-60-minute periods	-3.397	65.104	7.675	49.072	42.776	37.890	0.223	0.191	0.189
2	Est 2019-Tran 2018	110 Segment-60-minute periods	-10.199	62.234	0.748	47.546	41.186	36.915	0.219	0.193	0.173
	Est 2018-Tran 2019	88 Segment-60-minute periods	2.595	52.050	9.558	38.959	34.360	31.665	0.191	0.193	0.170
	Pooled (Est 2019- Tran 2018 and Est 2018-Tran 2019)	198 Segment-60-minute periods	-4.491	58.119	3.508	43.715	38.435	34.571	0.207	0.193	0.171
3	Est 2019-Tran 2018	110 Segment-60-minute periods	-14.323	66.591	1.017	50.324	45.668	37.563	0.219	0.176	0.183
	Est 2018-Tran 2019	88 Segment-60-minute periods	10.081	53.170	16.216	41.566	34.384	39.312	0.200	0.186	0.177
	Pooled (Est 2019- Tran 2018 and Est 2018-Tran 2019)	198 Segment-60-minute periods	-3.477	62.047	8.557	46.432	41.171	38.461	0.210	0.180	0.179

Table 3.9: Summary statistics of metrics obtained with Method 2 (arithmetic average of regression-based transformations of individual bus pass volumes) using SPEC 20 for 60-minute period estimates, by Case

Case Number	Estimation-Transformation	Number of Segment Direction Periods	Difference			ABS Difference			ARE		
			Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	Est 2019-Tran 2018	110 Segment-60-minute periods	-13.752	68.163	2.844	50.607	47.461	37.693	0.220	0.178	0.189
	Est 2018-Tran 2019	88 Segment-60-minute periods	10.068	59.103	12.395	43.851	40.630	36.462	0.208	0.206	0.179
	Pooled (Est 2019-Tran 2018 and Est 2018-Tran 2019)	198 Segment-60-minute periods	-3.166	65.225	7.650	47.604	44.572	37.031	0.215	0.190	0.180
2	Est 2019-Tran 2018	110 Segment-60-minute periods	-10.150	62.453	0.303	47.588	41.456	36.770	0.219	0.193	0.175
	Est 2018-Tran 2019	88 Segment-60-minute periods	3.001	51.813	9.678	39.127	33.838	33.200	0.194	0.201	0.165
	Pooled (Est 2019-Tran 2018 and Est 2018-Tran 2019)	198 Segment-60-minute periods	-4.283	58.175	6.388	43.813	38.383	34.012	0.208	0.197	0.168
3	Est 2019-Tran 2018	110 Segment-60-minute periods	-14.165	67.391	1.214	50.211	46.903	36.763	0.218	0.177	0.182
	Est 2018-Tran 2019	88 Segment-60-minute periods	10.517	53.895	15.907	42.167	34.900	35.904	0.205	0.207	0.169
	Pooled (Est 2019-Tran 2018 and Est 2018-Tran 2019)	198 Segment-60-minute periods	-3.195	62.824	7.027	46.636	42.085	36.723	0.212	0.191	0.173

Similar to the results obtained from Method 1, the results in Tables 3.6-3.9 show that the mean *differences* in the various data sets are relatively small compared to the standard deviations for both SPEC 1 and SPEC 20, in all Cases, for all datasets, and for both the 15-and 60-minute estimation periods. Therefore, there is no evidence that the video-based volume estimates obtained with Method 2 are either systematically greater than or less than the tube-based volumes. Moreover, similar to what was seen in Method 1, Case 2 is the best Case when pooling 2018 and 2019 segments. In addition, mean and median values of *ARE's* for 60-minute estimates are lower than those for 15-minute estimates, which is also compatible to Method 1. In Method 1, the magnitude of the mean and median of the *absolute differences* and *ARE's* in the 2018 data set are all lower than the values in the 2019 data set in all Cases and for both estimation periods. This pattern is the same in Method 2 for mean *ARE's* in 15-minute estimation, but not necessarily for median *ARE's* nor for mean and median values of *absolute differences* in 15-minute estimation period. This pattern does not also hold for the mean and median values of *absolute differences* and *ARE* metrics in 60-minute periods.

Comparing the advantage of using regression transformations to improve the individual bus pass estimates is not the focus of this thesis. The focus is to investigate different approaches to aggregating volumes obtained from the individual bus passes, as will be seen when comparing approaches in Chapter 4. Nevertheless, it is interesting to compare results from the arithmetic average aggregation approach using the bus-pass volumes directly obtained from Equation 2.3.1 (Method 1) to results from the arithmetic average aggregation approach after transforming these volumes by the regression equations (Method 2). To make these comparisons easier, Table 3.10 and 3.11 are provided. These tables, with values taken from previous tables, present the summary statistics of metrics for the pooled data sets for the three Cases obtained using each method in 15- and 60-minute estimation periods.

Table 3.10: Summary statistics of metrics obtained with Method 1 (arithmetic average of volume estimates from bus passes) and Method 2 (arithmetic average of regression-based transformation of volumes from bus passes) with the pooled (Est 2019- Tran 2018 and Est 2018-Tran 2019) Data sets for 15-minute estimation periods, by Case; Number of observations (segment-15 min periods) = 645

Case Number	**Method**	**Difference**			**ABS Difference**			**ARE**		
		Mean	**Standard Deviation**	**Median**	**Mean**	**Standard Deviation**	**Median**	**Mean**	**Standard Deviation**	**Median**
1	Method 1	6.332	28.876	2.962	19.704	22.027	13.149	0.355	0.365	0.268
	Method 2-SPEC 1	-0.689	18.326	1.885	13.898	11.952	11.264	0.268	0.240	0.210
	Method 2-SPEC 20	-0.607	18.458	1.719	13.736	12.332	10.874	0.263	0.239	0.203
2	Method 1	3.449	22.020	2.142	16.493	14.978	12.199	0.320	0.318	0.256
	Method 2-SPEC 1	-0.353	16.346	1.583	12.618	10.385	10.397	0.255	0.237	0.202
	Method 2-SPEC 20	-0.303	16.354	1.600	12.632	10.379	10.287	0.256	0.240	0.201
3	Method 1	4.505	23.733	2.906	17.677	16.451	12.916	0.333	0.326	0.260
	Method 2-SPEC 1	-0.701	17.783	1.699	13.517	11.564	10.847	0.259	0.231	0.209
	Method 2-SPEC 20	-0.652	17.971	1.796	13.568	11.791	10.662	0.260	0.239	0.206

Table 3.11: Summary statistics of metrics obtained with Method 1 (arithmetic average of volume estimates from bus passes) and Method 2 (arithmetic average of regression-based transformation of volumes from bus passes) with the pooled (Est 2019- Tran 2018 and Est 2018-Tran 2019) Data sets for 60-minute estimation periods, by Case; Number of observations (segment-15 min periods) = 198

Case Number	Method	Difference			ABS Difference			ARE		
		Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	Method 1	27.013	91.653	13.638	60.126	74.162	36.240	0.259	0.273	0.188
	Method 2-SPEC 1	-3.397	65.104	7.675	49.072	42.776	37.890	0.223	0.191	0.189
	Method 2-SPEC 20	-3.166	65.225	7.650	47.604	44.572	37.031	0.215	0.190	0.180
2	Method 1	13.867	63.202	11.616	46.262	45.128	33.418	0.221	0.235	0.154
	Method 2-SPEC 1	-4.491	58.119	3.508	43.715	38.435	34.571	0.207	0.193	0.171
	Method 2-SPEC 20	-4.283	58.175	6.388	43.813	38.383	34.012	0.208	0.197	0.168
3	Method 1	19.023	70.252	12.683	51.198	51.619	35.174	0.234	0.227	0.170
	Method 2-SPEC 1	-3.477	62.047	8.557	46.432	41.171	38.461	0.210	0.180	0.179
	Method 2-SPEC 20	-3.195	62.824	7.027	46.636	42.085	36.723	0.212	0.191	0.173

Looking at the summary metric values in Tables 3.10 and 3.11, the differences between the mean and median values obtained when using SPEC 1 and SPEC 20 in Method 2 are generally much less than the differences in these values obtained when using either of the Method 2 specifications and the values obtained when using Method 1. That is, the specification does not seem to be as important as the Method. Therefore, to facilitate comparisons, one specification is chosen for applications with regression-based Methods in Chapter 4. Whether SPEC 1 does better or worse than SPEC 20 appears to depend on the Case, the metric and whether mean or median is considered, and SPEC 20 is somewhat arbitrarily chosen for use in Chapter 4. The tables also indicate that, regardless of the specification, Method 2 generates systematically lower mean and median *absolute differences* and *ARE's* than Method 1. That is, transforming the volume estimates through the regression model leads to lower errors when used with the arithmetic aggregation approach, even though the transformed data were obtained from coefficients estimated on a different data set.

3.3. Summary of Results of Interest for Subsequent Comparisons

The following summarizes the findings in this chapter that will be of interest when evaluating the results obtained with different aggregation approaches presented in the next chapter. The magnitudes of the summary statistics of the metrics depend on Case, data set (segment), estimation period, and whether or not the bus pass volume estimates are transformed by a regression model. Therefore, results will be compared by these dimensions. In addition, the following results are obtained:

- There is no evidence of systematic over- or underestimation of period volumes when applying either Method.
- Case 2 is best among all three Cases for handling exceptionally high or low bus pass level volumes for both Methods.
- For both Methods, 60-minute estimates have lower mean and median *AREs* than have 15-minute estimates.
- Transforming the individual bus pass volume estimates through a regression model estimated on an independent data set leads to systematic lower *absolute differences* and *ARE*s compared to not transforming the bus pass estimates.

Consistency with these results will be examined when using other aggregation approaches in Chapter 4.

Finally, it was noticed that the actual form of the regression specification considered was not important in the empirical study conducted. Therefore, only one specification will be used for subsequent comparisons and as mentioned above, SPEC 20 is chosen.

Chapter 4: Aggregation of Single Bus Pass Volumes in Continuous Time

In Chapter 3, Method 1 aggregates volume estimates obtained from individual bus passes by taking the arithmetic average of the bus pass-derived volumes in intervals of interest. Method 2 also aggregates the individual bus-pass volumes using an arithmetic average of the bus pass volumes in the intervals of interest but conducts the aggregation after first transforming the bus pass volumes through a regression model. The intervals considered are discrete and nonoverlapping, which can lead to considering estimates farther apart in time to be considered more similar than estimates closer together in time. For example, consider a period with t_{beg} = 8:00 a.m. for $t_d = 15$ minutes and volume estimates from three bus passes occurring at 8:01 a.m., 8:14 a.m., and 8:15 a.m. The volume estimate from the 8:14 a.m. bus pass will be averaged with the volume estimate from the 8:01 a.m. pass, whereas the volume estimate from the 8:15 a.m. pass will be averaged in the next 15-minute interval, even though traffic conditions at 8:14 a.m. would likely be more similar to those occurring at 8:15 a.m. than those occurring at 8:01 a.m. To address this issue, an aggregation method that considers time of the estimates as a continuous variable is proposed and evaluated in this chapter. The general idea of this aggregation method is to integrate the flow rate vs time relation in intervals of interest. Perhaps the most straightforward way to implement this idea leads to the approach used for Methods 3 and 4 in this study, as presented in Section 4.2. A refinement of this approach smooths the flow rates from individual bus passes before integration and leads to the approach used for Methods 5 and 6 presented in Section 4.3.

4.1. Integration of Flow Rate Relation to Determine Period Volumes

In Figure 4.1, 1-minute bus-pass volumes (i.e., $T = 1$ in equation 2.3.1) on Segment 1.1 in 2018 are plotted against the time the bus entered the segment on the pass. The basis of the continuous time approaches presented in this thesis is that the 1-minute volumes are considered as flow rates in units of vehicles per unit time. Using $q(t)$ to denote the flow rate at time t, the volume in time period dt is $q(t)dt$, and the video-based volume $V_{t_d,t_{beg}}^{Video}$ can be found by integrating the $q(t)$ function from the beginning of the period until the end of the period:

$$V_{t_d,t_{beg}}^{Video} = \int_{t_{beg}}^{t_{beg}+t_d} q(t)dt \tag{4.1.1}$$

Since the integral of the $q(t)$ function is the area under the curve, $V_{t_d,t_{beg}}^{Video}$ can be found by finding the area under the $q(t)$ curve between t_{beg} and $t_{beg} + t_d$. In this thesis, the flow rates will continue to be considered in vehicles per minute and time will, therefore, be measured in minutes, although any consistent set of units can be used.

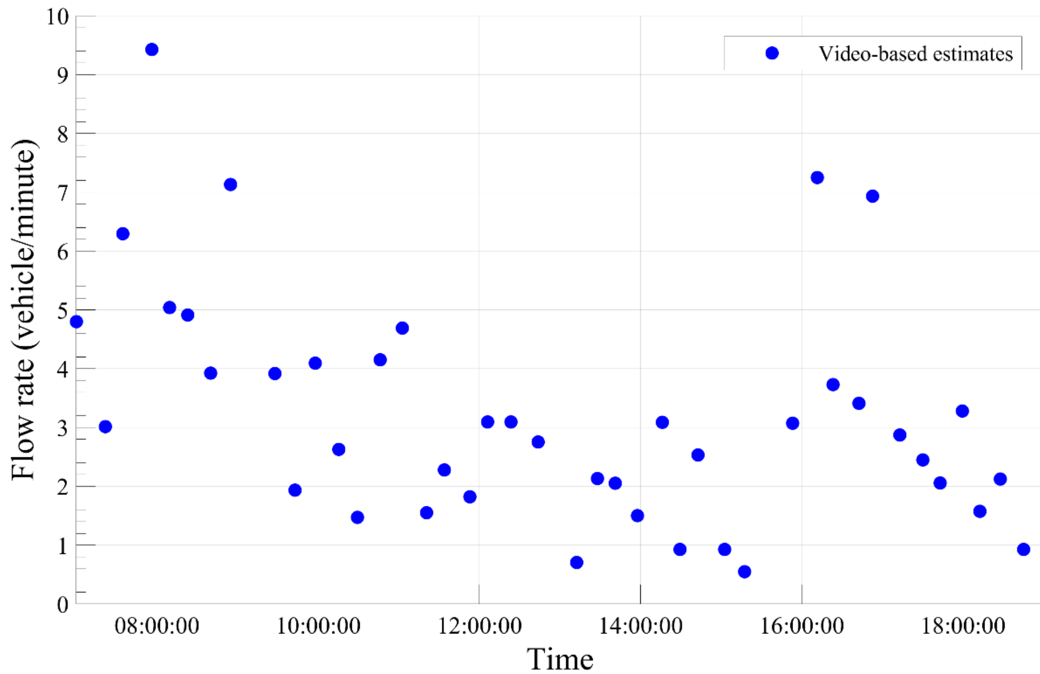

Figure 4.1: Example of flow rate concept: Vehicles per minute flow rates versus time t for Segment 1.1 in 2018

A straightforward way to implement the integral-benchmark method is to connect consecutive flow rates in time by line segments, as depicted in Figure 4.2. Then, the area under the curve between consecutive flow rates (corresponding to two consecutive bus passes) is the area of a trapezoid with base equal to the time between consecutive bus passes and heights equal to the flow rates (1-minute volumes) corresponding to each bus pass.

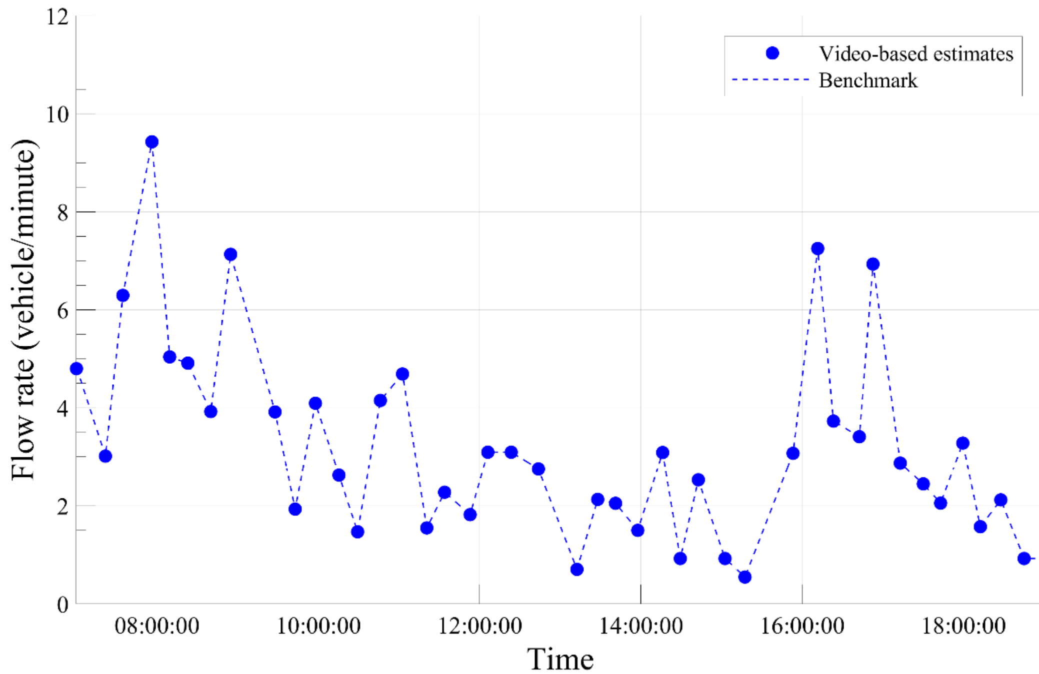

Figure 4.2: Illustration of interpolation between consecutive flow rates for Segment 1.1 in 2018

4.2. Integral-Benchmark Implementation of Continuous Time Approach

4.2.1. Method 3: Integral-Benchmark Approach with Flow Rates Derived Directly from Individual Bus Passes

Method 3 considers the area under the flow rate vs. time curve when using flow rates from individual bus passes determined directly form Equation 2.3.1 and the interpolation scheme illustrated in Figure 4.2. To investigate the accuracy of Method 3 relative to the tube volumes for the corresponding time interval, video-based volumes determined with this method are compared to the corresponding tube volumes to determine the *Difference*, *Absolute Difference*, and *ARE* (see Equations 3.1.1 – 3.1.3) for Cases 1-3 defined in Section 3.1.3. These metrics are calculated for the same estimation periods considered in Section 3.1.2, namely t_{beg} = 7:30, 7:45, …, and 18:15 for t_d = 15 (minutes) and t_{beg} = 7:30, 8:30, …, and 17:30 for t_d = 60 (minutes). All video-based volumes are used to fit the flow rate curve and allow the flow rates to be interpolated across the beginning and end of estimation periods. For example, if the first bus pass in the interval of [7:30, 7:45] occurred at 7:35 a.m., the flow rate needed at 7:30 a.m. (the beginning of the interval) is found by linearly interpolating between the flow rate at 7:35 and the flow rate corresponding to the last bus pass in the previous interval, [7:15, 7:30], even though metrics are not calculated for the [7:15, 7:30] interval. Finally, when determining distributions of the metrics, similar to what was done in Chapter 3, the intervals $[t_{beg}, t_{beg} + 15]$ on segments with no bus passes are not considered. That is, metrics for these intervals could be calculated, but to allow comparisons among different methods, they are not summarized in this thesis.

Summary statistics of the metrics obtained from Method 3 for 15-and 60-minute estimation periods are presented in Tables 4.1 and 4.2. The layout and notation of the tables are the same as those used in when presenting results in Method 1 in Tables 3.2-3.3. Summary statistics for each segment are presented in Appendix B.

Table 4.1: Summary statistics of metrics obtained with Method 3 (integral-benchmark approach applied to volume estimates directly obtained from individual bus passes), by Case and Data set, for 15-minute periods

Case Number	**Data set**	**Number of Segment Direction Periods**	**Difference**			**ABS Difference**			**ARE**		
			Mean	**Standard Deviation**	**Median**	**Mean**	**Standard Deviation**	**Median**	**Mean**	**Standard Deviation**	**Median**
1	2018 Pooled	370 Segment-15-minute periods	4.648	22.346	2.128	15.259	16.956	9.491	0.273	0.291	0.198
	2019 Pooled	275 Segment-15-minute periods	9.585	28.697	5.203	19.832	22.825	12.870	0.357	0.337	0.264
	2018 and 2019 Pooled	645 Segment-15-minute periods	6.722	25.312	3.034	17.184	19.754	11.026	0.308	0.314	0.233
2	2018 Pooled	347 Segment-15-minute periods	2.140	16.437	1.231	12.253	11.145	8.498	0.242	0.262	0.177
	2019 Pooled	268 Segment-15-minute periods	5.650	20.239	3.784	16.021	13.565	11.760	0.308	0.252	0.248
	2018 and 2019 Pooled	615 Segment-15-minute periods	3.646	18.233	2.327	13.870	12.373	10.254	0.271	0.260	0.208
3	2018 Pooled	370 Segment-15-minute periods	2.694	18.194	1.892	13.376	12.605	9.081	0.257	0.277	0.186
	2019 Pooled	275 Segment-15-minute periods	7.637	22.377	5.203	17.880	15.441	12.796	0.331	0.261	0.270
	2018 and 2019 Pooled	645 Segment-15-minute periods	4.777	20.195	3.024	15.274	14.037	10.912	0.288	0.273	0.220

Table 4.2: Summary statistics of metrics obtained with Method 3 (integral-benchmark approach applied to volume estimates directly obtained from individual bus passes), by Case and Data set, for 60-minute periods

Case Number	Data set	Number of Segment Direction Periods	Difference			ABS Difference			ARE		
			Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	2018 Pooled	110 Segment-60-minute periods	7.666	52.226	1.792	38.72	35.588	27.448	0.186	0.153	0.158
	2019 Pooled	88 Segment-60-minute periods	29.561	72.578	31.159	56.724	53.676	41.433	0.270	0.211	0.207
	2018 and 2019 Pooled	198 Segment-60-minute periods	17.397	62.790	13.035	46.720	45.249	34.029	0.224	0.185	0.174
2	2018 Pooled	110 Segment-60-minute periods	5.829	46.970	0.525	33.722	32.967	25.650	0.180	0.213	0.142
	2019 Pooled	88 Segment-60-minute periods	21.961	55.476	27.137	48.999	33.527	39.094	0.241	0.164	0.198
	2018 and 2019 Pooled	198 Segment-60-minute periods	13.056	51.379	11.452	40.566	33.952	32.530	0.208	0.194	0.159
3	2018 Pooled	110 Segment-60-minute periods	4.711	45.665	1.792	35.670	28.584	27.448	0.180	0.149	0.155
	2019 Pooled	88 Segment-60-minute periods	26.020	61.004	31.159	53.150	39.173	41.433	0.258	0.175	0.207
	2018 and 2019 Pooled	198 Segment-60-minute periods	14.182	53.866	13.035	43.439	34.673	34.029	0.214	0.165	0.171

Similar to what was seen when using Method 1 (and Method 2), Tables 4.1 and 4.2 show that the means of the *difference* metric for all data sets and Cases are relatively small compared to the standard deviations. Therefore, no striking over- or underestimation from the video data is present. As was seen with Method 1, segment-periods involved in 2019 generates higher mean and median *absolute differences* and *AREs* than segment-periods in 2018, in both 15-and 60-minute estimation periods and all three Cases. (This was not necessarily the same as the results obtained from Method 2.) Also, based on the summary statistics of pooled 2018 and 2019 data sets, Case 2 is the best case in both 15-and 60-minute estimation periods, which is compatible with the results of Method 1 (and Method 2). Finally, considering mean and median *ARE's* in pooled 2018 data set and pooled 2019 data set, 60-minute estimations generate lower errors compared to 15-minute estimations which is again the same as Method 1 (and Method 2).

To be able to compare the results obtained using Method 1 (arithmetic average of directly obtained volume estimates from bus passes) and Method 3 (integral-benchmark approach with flow rates obtained directly from individual bus passes), Tables 4.3 and 4.4 are provided. These Tables present the summary statistics of metrics for the pooled 2018 and 2019 data sets for the three Cases in 15-and 60-minute estimation periods.

Table 4.3: Summary statistics of metrics using Method 1 (arithmetic average approach) and Method 3 (integral-benchmark approach) with the pooled 2018 and 2019 Data sets for 15-minute estimation periods, by Case; Number of observations (segment-15 min periods) = 645

Case Number	**Method**	**Difference**			**ABS Difference**			**ARE**		
		Mean	**Standard Deviation**	**Median**	**Mean**	**Standard Deviation**	**Median**	**Mean**	**Standard Deviation**	**Median**
1	Method 1	6.332	28.876	2.962	19.704	22.027	13.149	0.355	0.365	0.268
	Method 3	6.722	25.312	3.034	17.184	19.754	11.026	0.308	0.314	0.233
2	Method 1	3.449	22.020	2.142	16.493	14.978	12.199	0.320	0.318	0.256
	Method 3	3.646	18.233	2.327	13.870	12.373	10.254	0.271	0.260	0.208
3	Method 1	4.505	23.733	2.906	17.677	16.451	12.916	0.333	0.326	0.260
	Method 3	4.777	20.195	3.024	15.274	14.037	10.912	0.288	0.273	0.220

Table 4.4: Summary statistics of metrics using Method 1 (arithmetic average approach) and Method 3 (integral-benchmark approach) with the pooled 2018 and 2019 Data sets for 60-minute estimation periods, by Case; Number of observations (segment-60 min periods) = 198

Case Number	**Method**	**Difference**			**ABS Difference**			**ARE**		
		Mean	**Standard Deviation**	**Median**	**Mean**	**Standard Deviation**	**Median**	**Mean**	**Standard Deviation**	**Median**
1	Method 1	27.013	91.653	13.638	60.126	74.162	36.240	0.259	0.273	0.188
	Method 3	17.397	62.790	13.035	46.720	45.249	34.029	0.224	0.185	0.174
2	Method 1	13.867	63.202	11.616	46.262	45.128	33.418	0.221	0.235	0.154
	Method 3	13.056	51.379	11.452	40.566	33.952	32.530	0.208	0.194	0.159
3	Method 1	19.023	70.252	12.683	51.198	51.619	35.174	0.234	0.227	0.170
	Method 3	14.182	53.866	13.035	43.439	34.673	34.029	0.214	0.165	0.171

Looking at the summarized metrics in Tables 4.3 and 4.4, mean and median *absolute difference* and *ARE* values are lower in Method 3 than in Method 1 for all three Cases when considering 15-minute estimates. This also holds for the mean and median *absolute difference* and mean *ARE* when considering 60-minute periods, However, the median *ARE* values in Cases 2 and 3 are slightly higher in Method 3 than in Method 1 for 60-minute periods. The changes in mean and median *absolute difference* and *ARE* also depend on the Case. Generally, however, when aggregating the video-based volumes obtained directly from Equation 2.3.1, Method 3 (benchmark integration approach) generates lower errors than Method 1 (arithmetic averaging approach).

4.2.2. Method 4: Integral-Benchmark Approach with Flow Rates from Individual Bus Passes after Regression-based Transformation

Method 4 uses the integral-benchmark approach to aggregate the flow rates obtained from individual bus passes in the intervals of interest but after transforming the bus pass volumes with a regression equation. For Method 4 (and Method 6), SPEC 20 (see Section 3.2.2.1) is used as the regression specification, and the estimated coefficients are those found in Section 3.2.2.2. Specifically, the coefficients estimated using the 2018 data are used to transform the 2019 bus pass volume, and the coefficients estimated using the 2019 data are used to transform the 2018 bus pass volumes. Similar to what was done to examine the accuracy of Methods 1-3, the video-based volumes determined with Method 4 are compared to the corresponding tube volumes to determine the *Difference*, *Absolute Difference*, and *ARE* (see Equations 3.1.1 – 3.1.3) for Cases 1-3 defined in Section 3.1.3, and for the same estimation periods considered in Section 3.1.2, i.e., periods beginning at

7:30 and ending at 18:15 for $t_d = 15$ (minutes) and ending at 17:30 for $t_d = 60$ (minutes) and for which there is at least one bus pass in the period.

Summary statistics of the metrics obtained from Method 4 for 15- and 60-minute estimation periods are presented in Tables 4.5 and 4.6. The layout and notation of the tables are the same as Tables 3.6-3.9 obtained from Method 2. Summary statistics for each segment are also presented in Appendix B.

Table 4.5: Summary statistics of metrics obtained with Method 4 (integral-benchmark aggregation of regression-based transformations), using SPEC 20 for 15-minute period estimates, by Case

Case Number	Estimation/ Transformation	Number of Segment Direction Periods	Difference			ABS Difference			ARE		
			Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	(Est 2019, Tran 2018)	370 Segment-15-minute periods	-3.326	18.364	-0.001	13.954	12.372	11.165	0.254	0.211	0.211
	(Est 2018, Tran 2019)	275 Segment-15-minute periods	3.560	16.235	3.977	12.135	11.335	8.952	0.253	0.258	0.180
	Pooled (Est 2019, Tran 2018) and (Est 2018, Tran 2019)	645 Segment-15-minute periods	-0.424	17.813	2.344	13.188	11.971	10.380	0.254	0.232	0.197
2	(Est 2018, Tran 2019)	347 Segment-15-minute periods	-1.997	15.848	0.598	12.510	9.910	10.398	0.246	0.212	0.197
	(Est 2018, Tran 2019)	268 Segment-15-minute periods	2.131	14.814	3.127	11.374	9.703	8.672	0.246	0.256	0.174
	Pooled (Est 2019, Tran 2018) and (Est 2018, Tran 2019)	615 Segment-15-minute periods	-0.226	15.536	2.232	12.022	9.830	9.712	0.246	0.231	0.188
3	(Est 2019, Tran 2018)	370 Segment-15-minute periods	-3.442	18.168	0.308	13.869	12.210	10.874	0.253	0.209	0.207
	(Est 2018, Tran 2019)	275 Segment-15-minute periods	3.609	15.036	4.281	11.867	9.889	9.327	0.251	0.258	0.176
	Pooled (Est 2019, Tran 2018) and (Est 2018, Tran 2019)	645 Segment-15-minute periods	-0.476	17.262	2.495	13.025	11.326	10.271	0.252	0.230	0.198

Table 4.6: Summary statistics of metrics obtained with Method 4 (integral-benchmark aggregation of regression-based transformations), using SPEC 20 for 60-minute period estimates, by Case

Case Number	Estimation/ Transformation	Number of Segment Direction Periods	Difference			ABS Difference			ARE		
			Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	(Est 2019, Tran 2018)	110 Segment-60-minute periods	-2.741	48.497	8.077	35.710	32.648	26.882	0.169	0.117	0.161
	(Est 2018, Tran 2019)	88 Segment-60-minute periods	17.650	39.047	19.745	34.501	25.098	27.598	0.179	0.148	0.138
	Pooled (Est 2019, Tran 2018) and (Est 2018, Tran 2019)	198 Segment-60-minute periods	6.321	45.526	15.586	35.173	29.426	27.126	0.174	0.131	0.153
2	(Est 2019, Tran 2018)	110 Segment-60-minute periods	0.016	44.131	7.823	34.845	26.748	24.621	0.176	0.142	0.141
	(Est 2018, Tran 2019)	88 Segment-60-minute periods	13.340	37.565	17.827	32.281	23.050	27.477	0.171	0.147	0.147
	Pooled (Est 2019, Tran 2018) and (Est 2018, Tran 2019)	198 Segment-60-minute periods	5.985	41.691	14.136	33.696	25.093	26.590	0.174	0.144	0.141
3	(Est 2019, Tran 2018)	110 Segment-60-minute periods	-3.340	48.232	7.135	35.676	32.347	27.003	0.169	0.117	0.152
	(Est 2018, Tran 2019)	88 Segment-60-minute periods	16.580	37.952	18.421	33.383	24.198	27.970	0.174	0.148	0.137
	Pooled (Est 2019, Tran 2018) and (Est 2018, Tran 2019)	198 Segment-60-minute periods	5.513	44.907	15.082	34.657	28.923	27.124	0.171	0.131	0.151

The summary statistics of mean and standard deviation of the *difference* metric in Tables 4.5 and 4.6 reveal that, similar to Method 2, the video-based estimates obtained from Method 4 do not systematically over- or underestimate the tube-based volumes, since the mean values for all data sets and Cases are relatively small compared to the standard deviations. Comparing the mean and median *ARE's* in the (Est 2019, Tran 2018) data set to the mean and median *ARE*'s in the (Est 2018, Tran 2019) data set shows that the effect of Cases is not as noticeable as when aggregating by arithmetic averaging in Method 2. However, Case 2 generates slightly lower mean ARE's than Cases 1 and 3, which is similar to what was seen in Method 2 (and Methods 1 and 3). The magnitudes of mean and median values of *ARE's* for 60-minute estimates are lower than those for 15-minute estimates for the corresponding data sets, which is compatible with the results obtained from Method 2 (and Methods 1 and 3). The values of the *absolute difference* and *ARE* metrics in the (Est 2019, Tran 2018) data set are higher than the values in the (Est 2018, Tran 2019) data set in all Cases in 15-minute estimation periods. However, in 60-minute estimation periods, there is no clear indication of which data set leads to better results, and the comparisons between the data sets depends on whether the mean or median is considered. In Method 2 (the previous Method that used regression-based transformation), the (Est 2018, Tran 2019) data set performed better than the (Est 2019, Tran 2018) data set for 60-minute estimation periods, and the relative performance of the two data sets for 15-minute estimation periods was slightly in favor of the (Est 2018, Tran 2019) data set but depended on which metrics were considered.

To compare results from arithmetic average of regression-based transformation (Method 2) to results from integral-benchmark aggregation of regression-based transformation (Method 4), Tables 4.7 and 4.8 are provided. These tables, with values taken from previous tables, present the summary statistics of metrics for the pooled (Est 2019, Tran 2018) and (Est 2018, Tran 2019) data sets for the three Cases obtained using each method in 15- and 60-minute estimation periods.

Table 4.7: Summary statistics of metrics obtained with Method 2 (arithmetic average of regression-based transformations) and Method 4 (integral-benchmark aggregation of regression-based transformations) with pooled 2018 and 2019 Data sets for 15-minute estimation periods, by Case; Number of observations (segment-15 min periods) = 645

Case Number	Method	Difference			ABS Difference			ARE		
		Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	Method 2	-0.607	18.458	1.719	13.736	12.332	10.874	0.263	0.239	0.203
	Method 4	-0.424	17.813	2.344	13.188	11.971	10.380	0.254	0.232	0.197
2	Method 2	-0.303	16.354	1.600	12.632	10.379	10.287	0.256	0.240	0.201
	Method 4	-0.226	15.536	2.232	12.022	9.830	9.712	0.246	0.231	0.188
3	Method 2	-0.652	17.971	1.796	13.568	11.791	10.662	0.260	0.239	0.206
	Method 4	-0.476	17.262	2.495	13.025	11.326	10.271	0.252	0.230	0.198

Table 4.8: Summary statistics of metrics obtained with Method 2 (arithmetic average of regression-based transformations) and Method 4 (integral-benchmark aggregation of regression-based transformation) with pooled 2018 and 2019 Data sets for 60-minute estimation periods, by Case; Number of observations (segment-60 min periods) = 198

Case Number	**Method**	**Difference**			**ABS Difference**			**ARE**		
		Mean	**Standard Deviation**	**Median**	**Mean**	**Standard Deviation**	**Median**	**Mean**	**Standard Deviation**	**Median**
1	Method 2	-3.166	65.225	7.650	47.604	44.572	37.031	0.215	0.190	0.180
	Method 4	6.321	45.526	15.586	35.173	29.426	27.126	0.174	0.131	0.153
2	Method 2	-4.283	58.175	6.388	43.813	38.383	34.012	0.208	0.197	0.168
	Method 4	5.985	41.691	14.136	33.696	25.093	26.590	0.174	0.144	0.141
3	Method 2	-3.195	62.824	7.027	46.636	42.085	36.723	0.212	0.191	0.173
	Method 4	5.513	44.907	15.082	34.657	28.923	27.124	0.171	0.131	0.151

Looking at the magnitude of mean and median *absolute differences* and *ARE's* in Tables 4.7 and 4.8, Method 4 generates systematically lower errors than Method 2 in all Cases and in both estimation periods. According to the summary statistics, transforming the video-based volumes through a regression model and aggregating them with integral-benchmark approach leads to lower errors than transforming the volumes through a regression model but aggregating them with arithmetic averaging approach.

As in Section 3.2.2.2, although not the main point of this thesis, it is interesting to compare results from an aggregation approach applied to volume estimates directly obtained from individual bus passes to results from the same aggregation approach applied to regression-based transformations. To make these comparisons for the integral-benchmark without transformation and with transformation (i.e., Method 3 and Method 4) easier, Table 4.9 and 4.10 are provided. These tables, with values taken from previous tables, present the summary statistics of metrics for the pooled data sets for the three Cases obtained using each method in 15- and 60-minute estimation periods.

Table 4.9: Summary statistics of metrics obtained with Method 3 (integral-benchmark approach applied to volume estimates directly obtained from individual bus passes) and Method 4 (integral-benchmark aggregation of regression-based transformations) with pooled 2018 and 2019 Data sets for 15-minute estimation periods, by Case; Number of observations (segment-15 min periods) = 645

Case Number	**Method**	**Difference**			**ABS Difference**			**ARE**		
		Mean	**Standard Deviation**	**Median**	**Mean**	**Standard Deviation**	**Median**	**Mean**	**Standard Deviation**	**Median**
1	Method 3	6.722	25.312	3.034	17.184	19.754	11.026	0.308	0.314	0.233
	Method 4	-0.424	17.813	2.344	13.188	11.971	10.380	0.254	0.232	0.197
2	Method 3	3.646	18.233	2.327	13.870	12.373	10.254	0.271	0.260	0.208
	Method 4	-0.226	15.536	2.232	12.022	9.830	9.712	0.246	0.231	0.188
3	Method 3	4.777	20.195	3.024	15.274	14.037	10.912	0.288	0.273	0.220
	Method 4	-0.476	17.262	2.495	13.025	11.326	10.271	0.252	0.230	0.198

Table 4.10: Summary statistics of metrics obtained with Method 3 (integral-benchmark approach applied to volume estimates directly obtained from individual bus passes) and Method 4 (integral-benchmark aggregation of regression-based transformations) with pooled 2018 and 2019 Data sets for 60-minute estimation periods, by Case; Number of observations (segment-60 min periods) = 198

Case Number	Method	Difference			ABS Difference			ARE		
		Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	Method 3	17.397	62.790	13.035	46.720	45.249	34.029	0.224	0.185	0.174
	Method 4	6.321	45.526	15.586	35.173	29.426	27.126	0.174	0.131	0.153
2	Method 3	13.056	51.379	11.452	40.566	33.952	32.530	0.208	0.194	0.159
	Method 4	5.985	41.691	14.136	33.696	25.093	26.590	0.174	0.144	0.141
3	Method 3	14.182	53.866	13.035	43.439	34.673	34.029	0.214	0.165	0.171
	Method 4	5.513	44.907	15.082	34.657	28.923	27.124	0.171	0.131	0.151

Looking at the summary metric values in Tables 4.9 and 4.10, Method 4 generates systematically lower mean and median *absolute differences* and *ARE's* than Method 3 for all Cases and estimation periods. That is, transforming the volume estimates through the regression model improves the quality of the results for the integral-benchmark approach, as it did when using the arithmetic average approach.

4.3. Smoothing of Flow Rate Curve for Implementation of Continuous Time Approach

The integral-benchmark approach used for Methods 3 and 4 interpolates between consecutive flow rates derived from each bus pass in continuous time (either without or with regression-based transformation), without consideration of the "noise" associated with volumes associated with an individual bus pass or of longer-term temporal trends in the data. Each bus pass estimate can be a noisy estimate of conditions at the time of the bus pass, and the time between consecutive bus passes can be small or large, compared to temporal changes in traffic conditions. In an attempt to address these issues when using the continuous time approach, the flow rates from the individual bus passes are "smoothed" in time. Many smoothing techniques could be considered. A "moving median time window" technique is considered in this thesis and explained in Section 4.3.1. The approach is implemented with volumes obtained directly from individual bus passes (Method 5) and with bus pass volumes after transformation with a regression model (Method 6) in Sections 4.3.2 and 4.3.3, respectively.

4.3.1. Moving Median Time Window

What is referred to in this thesis as the "moving median time window technique" considers window of time t_w and a step size t_s with both in the same units of time. The approach begins at time t_{w-beg} (e.g., t_{w-beg} = 7:15 in the empirical studies in this thesis so that there will be a point at the 7:30 a.m. which is the beginning of the first interval for which metrics are computed), determines the median of all the flow rates occurring in the interval $[t_{w-beg} + t_W]$, and considers this median value to occur at the midpoint of the window, i.e., at $\left[t_{w-beg} + \frac{t_W}{2}\right]$. Then, the window t_w is advanced by the step size t_s, and the process is repeated with the new t_{w-beg} equal to the t_{w-beg} used at the previous step plus t_s (e.g., $t_{w-beg} = 7{:}16$ in the example). This approach ends when the iterated value of t_{w-beg} is such that $\left[t_{w-beg} + \frac{t_W}{2}\right]$ is at the end of the last estimation period for which metrics are computed (e.g., t_{w-beg} = 6: 15 p.m. in the empirical studies in this thesis.) In this thesis, a window of t_w = 30 minutes and step size of t_s = 1 minute are chosen. Exploring other window times and step sizes could be a topic for future research.

As an example, the curve resulting from smoothing the flow rates determined directly for individual bus passes on Segment 1.1 (see Figure 4.1) with these parameters is depicted in red in Figure 4.3. In this figure, the blue curve represents the benchmark interpolation between flow rates used in Method 3. After smoothing the bus pass flow rates in this way, $V_{t_d,t_{beg}}^{Video}$ can again be found by integrating the smoothed $q(t)$ function between t_{beg} and $t_{beg} + t_d$, which is equivalent to finding the area under the curve between t_{beg} and $t_{beg} + t_d$. Once again, the area under the smoothed curve between smoothed flow rates is the area of the trapezoid between consecutive smoothed points.

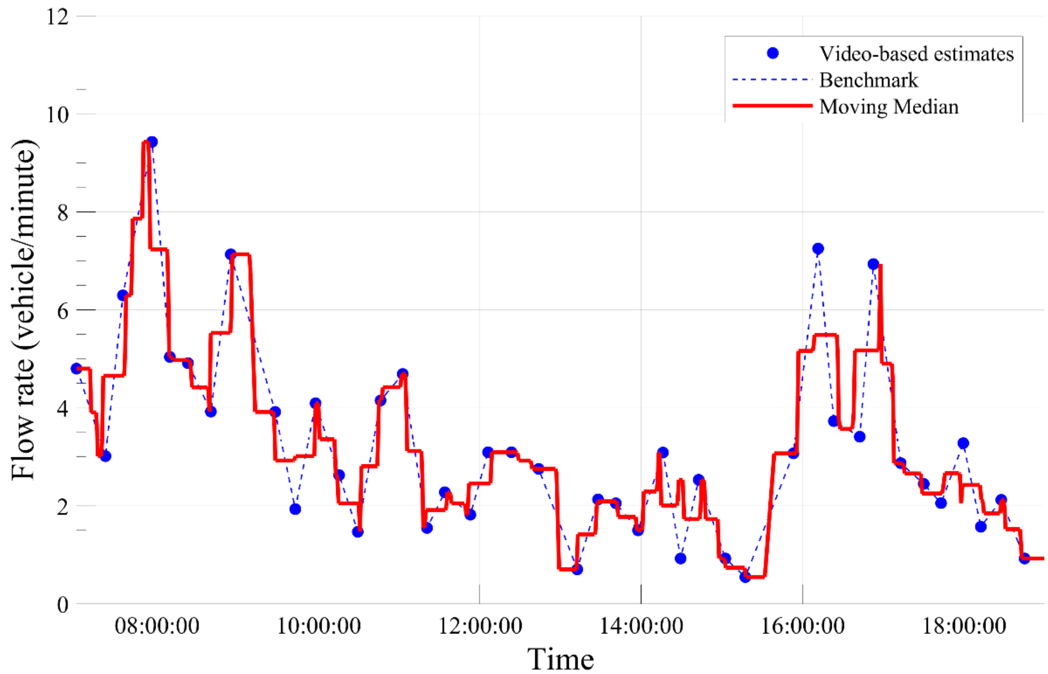

Figure 4.3: Example of smoothing flow rates in Figure 4.1.1 by using moving median time window technique with t_w = 30 minutes, t_s = 1 minute

4.3.2. Method 5: Smoothed Curve Approach with Flow Rates Derived Directly from Individual Bus Passes

Method 5 determines the areas under the smoothed flow rate curve, where the bus pass flow rates to be smoothed are obtained directly from Equation 2.3.1. Similar to previous aggregation methods, the results obtained from Method 5 are compared against road tube volumes at the corresponding time intervals and *Difference*, *Absolute Difference*, and *ARE* (see Equations 3.1.1 – 3.1.3) for Cases 1-3 (seen Section 3.1.3) are determined. These metrics are calculated for the same estimation periods considered in Section 3.1.2 (i.e., periods beginning at 7:30 and ending at 18:15 for $t_d = 15$ (minutes) and ending at 17:30 for $t_d = 60$ (minutes) and for which there is at least one bus pass in the period).

Summary statistics of the metrics obtained from Method 5 are presented in Table 4.11 and Table 4.12 for 15-and 60-minutes estimation periods, respectively. The layout and notation of the table is the same as that used in the corresponding table for Method 1 presented in Section 3.2.1. Summary statistics for each segment are presented in Appendix B.

Table 4.11: Summary statistics of metrics obtained with Method 5 (smoothed interpolation approach using volume estimates directly obtained from individual bus passes) by Case and Data set, for 15-minute periods

Case Number	Data set	Number of Segment Direction Periods	Difference			ABS Difference			ARE		
			Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	2018 Pooled	370 Segment-15-minute periods	4.032	22.564	1.381	15.387	16.972	10.003	0.278	0.288	0.209
	2019 Pooled	275 Segment-15-minute periods	9.496	28.583	5.140	19.245	23.147	12.776	0.344	0.339	0.263
	2018 and 2019 Pooled	645 Segment-15-minute periods	6.335	25.399	2.711	17.013	19.885	11.269	0.306	0.312	0.227
2	2018 Pooled	347 Segment-15-minute periods	1.415	16.891	0.795	12.483	11.448	9.420	0.249	0.265	0.184
	2019 Pooled	268 Segment-15-minute periods	5.502	20.073	4.069	15.339	14.040	11.613	0.293	0.252	0.229
	2018 and 2019 Pooled	615 Segment-15-minute periods	3.168	18.420	1.860	13.708	12.694	10.227	0.268	0.260	0.206
3	2018 Pooled	370 Segment-15-minute periods	2.108	18.445	1.170	13.515	12.709	9.738	0.263	0.276	0.196
	2019 Pooled	275 Segment-15-minute periods	7.809	22.970	5.061	17.547	16.727	12.740	0.322	0.271	0.261
	2018 and 2019 Pooled	645 Segment-15-minute periods	4.511	20.650	2.553	15.214	14.662	11.000	0.287	0.275	0.217

Table 4.12: Summary statistics of metrics obtained with Method 5 (smoothed interpolation approach using volume estimates directly obtained from individual bus passes) by Case and Data set, for 60-minute periods

Case Number	Data set	Number of Segment Direction Periods	Difference			ABS Difference			ARE		
			Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	2018 Pooled	110 Segment-60-minute periods	7.870	50.859	1.999	38.96	33.308	31.455	0.189	0.142	0.155
	2019 Pooled	88 Segment-60-minute periods	30.848	82.961	24.984	59.541	65.130	42.278	0.278	0.245	0.198
	2018 and 2019 pooled	198 Segment-60-minute periods	18.082	67.732	13.889	48.108	50.835	35.541	0.228	0.199	0.175
2	2018 Pooled	110 Segment-60-minute periods	5.756	48.360	-1.565	34.909	33.701	25.636	0.188	0.215	0.136
	2019 Pooled	88 Segment-60-minute periods	19.972	56.845	20.084	48.641	35.049	37.869	0.239	0.171	0.183
	2018 and 2019 Pooled	198 Segment-60-minute periods	12.125	52.594	10.635	41.061	34.853	34.005	0.210	0.197	0.155
3	2018 Pooled	110 Segment-60-minute periods	4.949	44.582	1.999	35.960	26.468	28.802	0.183	0.140	0.144
	2019 Pooled	88 Segment-60-minute periods	27.945	71.126	24.984	56.615	50.897	42.278	0.268	0.205	0.198
	2018 and 2019 Pooled	198 Segment-60-minute periods	15.170	58.776	13.889	45.140	40.407	35.541	0.220	0.176	0.172

From Tables 4.11 and 4.12, similar to what was found with Methods 1 and 3 (and Methods 2 and 4), the mean *differences* are relatively small compared to the standard deviations. Therefore, no striking over- or underestimation from the video data is present when using Method 5. As also seen in Methods 1 and 3 (but not necessarily in Methods 2 and 4), the magnitudes of the mean and median *absolute differences* and *ARE's* in the pooled 2019 data set in both 15- and 60-minute estimation periods and in all three Cases are higher than in the pooled 2018 data set. From the summary statistics of the pooled 2018 and 2019 data sets in both 15- and 60-minute estimation periods, one sees that Case 2 is better than Case 3, which is better than Case 1. This result is also compatible with the results from Methods 1 and 3 (and similar to Method 2 and Method 4). Comparing mean and median *ARE's* in 2018 and 2019 pooled data sets reveals that the 60-minute values are less than the 15-minute values. That is, 60-minute estimations generate lower relative errors, which is again the same result obtained with Methods 1 and 3 (and Methods 2 and 4).

To be able to compare the results obtained using the three aggregation approaches using directly obtained bus pass volumes (i.e., Methods 1, 3, and 5), Tables 4.13 and 4.14 are provided. These tables present the summary statistics of metrics for the pooled 2018 and 2019 data sets for the three Cases in 15- and 60-minute estimation periods.

Table 4.13: Summary statistics of metrics obtained with Methods using directly obtained bus pass volumes (Methods 1, 3, 5) with pooled with pooled 2018 and 2019 Data sets for 15-minute estimation periods, by Case; Number of observations (segment-15 min periods) = 645

Case Number	Method	Difference			ABS Difference			ARE		
		Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	Method 1	6.332	28.876	2.962	19.704	22.027	13.149	0.355	0.365	0.268
	Method 3	6.722	25.312	3.034	17.184	19.754	11.026	0.308	0.314	0.233
	Method 5	6.335	25.399	2.711	17.013	19.885	11.269	0.306	0.312	0.227
2	Method 1	3.449	22.020	2.142	16.493	14.978	12.199	0.320	0.318	0.256
	Method 3	3.646	18.233	2.327	13.870	12.373	10.254	0.271	0.260	0.208
	Method 5	3.168	18.420	1.860	13.708	12.694	10.227	0.268	0.260	0.206
3	Method 1	4.505	23.733	2.906	17.677	16.451	12.916	0.333	0.326	0.260
	Method 3	4.777	20.195	3.024	15.274	14.037	10.912	0.288	0.273	0.220
	Method 5	4.511	20.650	2.553	15.214	14.662	11.000	0.287	0.275	0.217

Table 4.14: Summary statistics of metrics obtained with Methods using directly obtained bus pass volumes (Methods 1, 3, 5) with pooled 2018 and 2019 Data sets for 60-minute estimation periods, by Case; Number of observations (segment-60 min periods) = 198

Case Number	Method	Difference			ABS Difference			ARE		
		Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	Method 1	27.013	91.653	13.638	60.126	74.162	36.240	0.259	0.273	0.188
	Method 3	17.397	62.790	13.035	46.720	45.249	34.029	0.224	0.185	0.174
	Method 5	18.082	67.732	13.889	48.108	50.835	35.541	0.228	0.199	0.175
2	Method 1	13.867	63.202	11.616	46.262	45.128	33.418	0.221	0.235	0.154
	Method 3	13.056	51.379	11.452	40.566	33.952	32.530	0.208	0.194	0.159
	Method 5	12.125	52.594	10.635	41.061	34.853	34.005	0.210	0.197	0.155
3	Method 1	19.023	70.252	12.683	51.198	51.619	35.174	0.234	0.227	0.170
	Method 3	14.182	53.866	13.035	43.439	34.673	34.029	0.214	0.165	0.171
	Method 5	15.170	58.776	13.889	45.140	40.407	35.541	0.220	0.176	0.172

Looking at Tables 4.13 and 4.14, one sees that the mean values of *absolute difference* and *ARE* metrics from Methods 3 and 5 are close to each other and they are lower than those from Method 1 for all Cases and both 15 – and 60-minute estimation periods. Although the mean values are close to each other, Method 5 performs systematically better than Method 3 for 15-minute estimation periods, and Method 3 performs systematically better than Method 5 in 60-minute estimation periods. When considering medians, Methods 3 and 5 are also close to each other and lower than Method 1 for 15-minute estimation periods but not necessarily for 60-minute estimation periods. When considering Case 1, Methods 3 and 5 are close and noticeably better than Method 1. When considering Case 2 Method 1 and 5 are close and slightly better than Method 3. When considering Case 3, all three Methods have similar medians. In conclusion, considering all the summary statistics, Cases, and estimation periods the integration approach does noticeably better than the arithmetic average approach. Whether Method 3 (integration with benchmark) does better than Method 5 (integration after smoothing) depends on the estimation period and the Case, and the summary statistics considered.

4.3.3. Method 6: Smoothed Curve Approach with Flow Rates from Individual Bus Passes after Regression-based Transformation

In Method 6, the bus pass flow rates are first transformed through a regression model. In this study, SPEC 20 (see Section 3.2.2.1) is used as the regression specification for Method 6, and the estimated coefficients are those found in Section 3.2.2.2. Specifically, identical to what was done with Method 4, the same coefficients estimated using the 2018 data are used to transform the same 2019 bus pass volumes, and the same coefficients estimated using the 2019 data are used to transform the same 2018 bus pass volumes. These transformed flow rates are then smoothed with the moving median time technique used in Section 4.3.1 and with the same window size (t_w = 30 minutes) and step size (t_s= 1 minute) used there. Then the areas under the smoothed flow rate curve are determined. Similar to what was done to examine the accuracy of Methods 1-5, the video-based volumes determined with Method 6 are compared to the corresponding tube volumes to determine the *Difference*, *Absolute Difference*, and *ARE* (see Equations 3.1.1 – 3.1.3) for Cases 1-3 defined in Section 3.1.3, and for the same estimation periods considered in Section 3.1.2, i.e., periods beginning at 7:30 and ending at 18:15 for $t_d = 15$ (minutes) and ending at 17:30 for $t_d = 60$ (minutes) and for which there is at least one bus pass in the period.

Summary statistics of the metrics obtained from Method 6 for 15- and 60-minute estimation periods are presented in Tables 4.15 and 4.16. The layout and notation of the tables are the same as the table used when evaluating Method 2 and Method 4. Summary statistics for each segment are also presented in Appendix B.

Table 4.15: Summary statistics of metrics obtained with Method 6 (smoothed interpolation aggregation of regression-based transformations) using SPEC 20 for 15-minute period estimates, by Case

Case Number	Estimation/ Transformation	Number of Segment Direction Periods	Difference			ABS Difference			ARE		
			Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	(Est 2019, Tran 2018)	370 Segment-15-minute periods	-3.464	18.454	0.156	14.097	12.382	10.876	0.256	0.210	0.214
	(Est 2018, Tran 2019)	275 Segment-15-minute periods	3.537	16.150	3.382	11.954	11.400	8.839	0.248	0.253	0.179
	Pooled (Est 2019, Tran 2018) and (Est 2018, Tran 2019)	645 Segment-15-minute periods	-0.513	17.845	2.058	13.194	12.015	10.438	0.253	0.229	0.199
2	(Est 2018, Tran 2019)	347 Segment-15-minute periods	-2.266	16.111	0.749	12.805	10.014	10.484	0.250	0.210	0.205
	(Est 2018, Tran 2019)	268 Segment-15-minute periods	2.024	14.548	2.846	11.073	9.627	8.535	0.240	0.251	0.172
	Pooled (Est 2019, Tran 2018) and (Est 2018, Tran 2019)	615 Segment-15-minute periods	-0.426	15.593	1.707	12.062	9.879	9.720	0.246	0.229	0.192
3	(Est 2019, Tran 2018)	370 Segment-15-minute periods	-3.625	18.292	0.193	14.041	12.251	10.924	0.255	0.208	0.214
	(Est 2018, Tran 2019)	275 Segment-15-minute periods	3.628	15.308	3.446	11.818	10.362	9.398	0.248	0.257	0.177
	Pooled (Est 2019, Tran 2018) and (Est 2018, Tran 2019)	645 Segment-15-minute periods	-0.568	17.457	2.017	13.104	11.537	10.256	0.252	0.230	0.200

Table 4.16: Summary statistics of metrics obtained with Method 6 (smoothed interpolation aggregation of regression-based transformations) using SPEC 20 for 60-minute period estimates, by Case

Case Number	Estimation/ Transformation	Number of Segment Direction Periods	Difference			ABS Difference			ARE		
			Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	(Est 2019, Tran 2018)	110 Segment-60-minute periods	-2.340	47.982	7.572	35.072	32.557	29.069	0.166	0.116	0.152
	(Est 2018, Tran 2019)	88 Segment-60-minute periods	18.314	42.394	20.888	36.119	28.468	30.828	0.184	0.151	0.143
	Pooled (Est 2019, Tran 2018) and (Est 2018, Tran 2019)	198 Segment-60-minute periods	6.840	46.560	16.355	35.537	30.691	29.824	0.174	0.133	0.147
2	(Est 2019, Tran 2018)	110 Segment-60-minute periods	0.036	44.078	9.467	34.634	26.939	26.530	0.175	0.143	0.144
	(Est 2018, Tran 2019)	88 Segment-60-minute periods	12.196	38.335	18.596	32.909	22.766	30.542	0.173	0.145	0.144
	Pooled (Est 2019, Tran 2018) and (Est 2018, Tran 2019)	198 Segment-60-minute periods	5.483	41.882	12.337	33.862	25.072	30.455	0.174	0.143	0.144
3	(Est 2019, Tran 2018)	110 Segment-60-minute periods	-3.123	47.797	5.270	35.147	32.267	30.141	0.166	0.117	0.150
	(Est 2018, Tran 2019)	88 Segment-60-minute periods	17.565	42.542	19.786	35.478	29.024	30.301	0.180	0.153	0.145
	Pooled (Est 2019, Tran 2018) and (Est 2018, Tran 2019)	198 Segment-60-minute periods	6.072	46.518	13.067	35.294	30.748	30.301	0.172	0.134	0.147

Looking at Tables 4.15 and 4.16, similar to the previous regression-based transformation Methods (i.e., Methods 2 and 4) and the non-regression-based Methods (i.e., Methods, 1, 2, and 5), the mean values for all data sets and Cases are relatively small compared to the standard deviations. Therefore, there is no evidence that the video-based volume estimates obtained with Method 6 are either systematically greater than or less than the tube-based volumes. Considering the mean and median *ARE's* in the (Est 2019, Tran 2018) and (Est 2018, Tran 2019) data sets, similar to Method 4, the effect of Cases is not as noticeable as when considering arithmetic averaging in Method 2. However, Case 2 generates slightly lower errors than Cases 1 and 3, which is similar to what was seen in Method 4. Moreover, similar to Methods 2 and 4 (and Methods, 1, 3, and 5), mean and median values of *ARE's* in 60-minute estimates are lower than those for 15-minute estimates for the corresponding data sets. The values of *absolute difference* and *ARE* metrics in the (Est 2019, Tran 2018) data set are higher than the values of the (Est 2018, Tran 2019) data set in all Cases in 15-minute estimation periods. This pattern was seen when using Method 4 but not necessarily when using Method 2. Moreover, in 60-minute estimation periods, there is no clear indication of which data set does better, and the performance of data sets depends on the metrics as it did in Method 4. In general, when comparing the two data sets, Methods 4 and 6 produce the same pattern for both 15 – and 60-minute estimation periods, and these patterns are slightly different than those seen in Method 2. The regression-based transformation seems to affect the two data sets differently.

To compare results from all three aggregation methods with regression-based transformation (i.e., Methods 2, 4, and 6), Tables 4.17 and 4.18 are provided. These tables, with values taken from previous tables, present the summary statistics of metrics for the pooled (Est 2019, Tran 2018) and (Est 2018, Tran 2019) data sets for the three Cases obtained using each method in 15- and 60-minute estimation periods.

Table 4.17: Summary statistics of metrics obtained with Methods using directly obtained bus pass volumes after transforming them through a regression model (Methods 2, 4, 6) with the pooled 2018 and 2019 Data sets for 15-minute estimation periods, by Case; Number of observations (segment-15 min periods) = 645

Case Number	Method	Difference			ABS Difference			ARE		
		Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	Method 2	-0.607	18.458	1.719	13.736	12.332	10.874	0.263	0.239	0.203
	Method 4	-0.424	17.813	2.344	13.188	11.971	10.380	0.254	0.232	0.197
	Method 6	-0.513	17.845	2.058	13.194	12.015	10.438	0.253	0.229	0.199
2	Method 2	-0.303	16.354	1.600	12.632	10.379	10.287	0.256	0.240	0.201
	Method 4	-0.226	15.536	2.232	12.022	9.830	9.712	0.246	0.231	0.188
	Method 6	-0.426	15.593	1.707	12.062	9.879	9.720	0.246	0.229	0.192
3	Method 2	-0.652	17.971	1.796	13.568	11.791	10.662	0.260	0.239	0.206
	Method 4	-0.476	17.262	2.495	13.025	11.326	10.271	0.252	0.230	0.198
	Method 6	-0.568	17.457	2.017	13.104	11.537	10.256	0.252	0.230	0.200

Table 4.18: Summary statistics of metrics obtained with Methods using directly obtained bus pass volumes after transforming them through a regression model (Methods 2, 4, 6) with the pooled 2018 and 2019 Data sets for 15-minute estimation periods, by Case; Number of observations (segment-60 min periods) = 198

Case Number	Method	Difference			ABS Difference			ARE		
		Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	Method 2	-3.166	65.225	7.650	47.604	44.572	37.031	0.215	0.190	0.180
	Method 4	6.321	45.526	15.586	35.173	29.426	27.126	0.174	0.131	0.153
	Method 6	6.840	46.560	16.355	35.537	30.691	29.824	0.174	0.133	0.147
2	Method 2	-4.283	58.175	6.388	43.813	38.383	34.012	0.208	0.197	0.168
	Method 4	5.985	41.691	14.136	33.696	25.093	26.590	0.174	0.144	0.141
	Method 6	5.483	41.882	12.337	33.862	25.072	30.455	0.174	0.143	0.144
3	Method 2	-3.195	62.824	7.027	46.636	42.085	36.723	0.212	0.191	0.173
	Method 4	5.513	44.907	15.082	34.657	28.923	27.124	0.171	0.131	0.151
	Method 6	6.072	46.518	13.067	35.294	30.748	30.301	0.172	0.134	0.147

Looking at Tables 4.17 and 4.18, the mean and median values of *absolute difference* and *ARE* metrics from Methods 4 and 6 are either the same or close to each other, and they are lower than these values from Method 2 in both 15- and 60-minute estimation periods. Comparing these values in Methods 4 and 6 in both 15- and 60-minute estimation periods reveals that Method 6 does better than Method 4 on a few comparisons, and Method 4 does better than Method 6 on slightly more comparisons, but the performances of the two methods is similar. In conclusion, using regression-based transformations in the integration-based aggregation approaches (Methods 4 and 6) does better than the using the regression-based transformations in the arithmetic average aggregation approach (Method 2). Whether Method 4 (integral-benchmark aggregation of regression-based transformations) does better than Method 6 (smoothed interpolation aggregation of regression-based transformation) using the window and step size values preset for this thesis depends on the estimation period, the Case, and the summary statistics considered.

Finally, as before, it is interesting to compare results from an aggregation approach applied to volume estimates directly obtained from individual bus passes to results from the same aggregation approach applied to regression-based transformations. To make these comparisons for the smoothed interpolation approach without transformation and with transformation (i.e., Method 5 to Method 6) easier, Table 4.19 and 4.20 are provided. These tables, with values taken from previous tables, present the summary statistics of metrics for the pooled data sets for the three Cases obtained using each method in 15- and 60-minute estimation periods.

Table 4.19: Summary statistics of metrics obtained with Method 5 (smoothed interpolation approach using volume estimates directly obtained from individual bus passes) and Method 6 (smoothed interpolation aggregation of regression-based transformation) with pooled 2018 and 2019 Data sets for 15-minute estimation periods, by Case; Number of observations (segment-15 min periods) = 645

Case Number	Method	Difference			ABS Difference			ARE		
		Mean	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
1	Method 5	6.335	25.399	2.711	17.013	19.885	11.269	0.306	0.312	0.227
	Method 6	-0.513	17.845	2.058	13.194	12.015	10.438	0.253	0.229	0.199
2	Method 5	3.168	18.420	1.860	13.708	12.694	10.227	0.268	0.260	0.206
	Method 6	-0.426	15.593	1.707	12.062	9.879	9.720	0.246	0.229	0.192
3	Method 5	4.511	20.650	2.553	15.214	14.662	11.000	0.287	0.275	0.217
	Method 6	-0.568	17.457	2.017	13.104	11.537	10.256	0.252	0.230	0.200

Table 4.20: Summary statistics of metrics obtained with Method 5 (smoothed interpolation approach using volume estimates directly obtained from individual bus passes) and Method 6 (smoothed interpolation aggregation of regression-based transformation) with pooled 2018 and 2019 Data sets for 60-minute estimation periods, by Case; Number of observations (segment-60 min periods) = 198

Case Number	**Method**	**Difference**			**ABS Difference**			**ARE**		
		Mean	**Standard Deviation**	**Median**	**Mean**	**Standard Deviation**	**Median**	**Mean**	**Standard Deviation**	**Median**
1	Method 5	18.082	67.732	13.889	48.108	50.835	35.541	0.228	0.199	0.175
	Method 6	6.840	46.560	16.355	35.537	30.691	29.824	0.174	0.133	0.147
2	Method 5	12.125	52.594	10.635	41.061	34.853	34.005	0.210	0.197	0.155
	Method 6	5.483	41.882	12.337	33.862	25.072	30.455	0.174	0.143	0.144
3	Method 5	15.170	58.776	13.889	45.140	40.407	35.541	0.220	0.176	0.172
	Method 6	6.072	46.518	13.067	35.294	30.748	30.301	0.172	0.134	0.147

Looking at the summary metric values in Tables 4.19 and 4.20, Method 6 generates systematically lower mean and median *absolute differences* and *ARE's* than Method 5. That is, transforming the volume estimates through the regression model improves the quality of the results for the smoothing approach, as it did when using the integral-benchmark and arithmetic average approach.

4.4. Summary of Results

The following summarizes the findings from the evaluations of the different aggregation approaches presented in this chapter and Chapter 3. In all Methods, the magnitudes of the summary statistics of the metrics depend on Case, data set (set of segments), estimation period, and whether or not the bus pass volume estimates are transformed by a regression model. In addition, the following results are obtained for all Methods:

- There is no evidence of systematic over- or underestimation of period volumes.
- Lower mean and median *ARE*s are obtained for 60-minute estimates than for 15-minute estimates.
- Transforming the individual bus pass volume estimates through a regression model estimated on an independent data set leads to systematic lower *absolute differences* and *ARE*s compared to not transforming the bus pass estimates.

Finally, it was noticed that aggregating video-based volumes (with or without transforming them through a regression model) in continuous time intervals (Methods 3 and 5, or Methods 4 and 6) leads to systematically lower errors than aggregating them in discrete time intervals (Method 1 or Method 2).

Chapter 5: Summary and Discussion

5.1. Summary and Conclusion

Using video imagery from transit buses to estimate traffic volumes on roadway segments can expand spatial coverage of data collection across urban networks at low marginal cost. Buses already cover major roads in urban networks to provide mobility service to the urban population, and many of these buses are already equipped with cameras for other purposes. Therefore, the platforms that allow spatial coverage and the sensors that allow video collection are already available. The method developed elsewhere to convert the vehicles appearing in the video imagery for one bus traversal of a roadway segment (i.e., for one "bus pass") to an individual bus pass volume was described in Chapter 2. As explained there, these "individual bus pass volumes" correspond to observations over very short time intervals, and any one bus pass volume would likely not be representative of the volume during the longer period of interest. However, buses repeatedly cover the same roadway segment, and one can conceivably aggregate the multiple individual bus pass volumes into a more representative estimate of the volume during the period of interest. This thesis focusses on this aggregation of the multiple volume estimates obtained from individual bus passes.

Several approaches to aggregate the volumes derived from the individual bus passes are proposed and evaluated in this thesis. These approaches consider the volumes of individual bus passes either to fall in "discrete" estimation periods, with no influence of volumes outside, but near, the period, or to be considered samples of a flow rate function that is “continuous” in time and which can be integrated to determine the estimated volume. The “arithmetic average” aggregation approach estimates the period level volume as the arithmetic average of the individual bus pass volumes across all bus passes contained in the discrete time interval forming the estimation period. The “integral-benchmark interpolation” approach uses a simple linear interpolation between consecutive individual bus pass volumes to determine the continuous flow rate function. The “integrated smoothed interpolation” approach smooths the individual bus pass volumes with a “moving median time window” technique to determine the flow rate function before integrating over time. To evaluate the different aggregation approaches, estimates obtained from each method using empirical video imagery collected from Campus Area Bus Service buses serving The Ohio State University are compared to traffic volumes obtained from concurrently collected road tube data provided by the Mid-Ohio Regional Planning Commission. Evaluations are conducted for 15- and 60-minute estimation periods. It is noted that video imagery can lead to what are considered “exceptionally low” or “exceptionally high” volume estimates on individual bus passes. To handle these exceptionally low or high volumes, three “Cases” are defined. In addition, two “Data Variants” of the volumes obtained from individual bus passes are used in the empirical evaluations. Data Variant 1 uses the volumes directly obtained from individual bus passes. Data Variant 2 uses a transformation of the Data Variant 1 bus pass volumes, where each volume is transformed through a regression model. When considering the three approaches and two Data Variants, a total of six “Methods” are investigated. Table 5.1, which is a reproduction of Table 1.1, summarizes the approaches, Data Variants, and Methods considered in this thesis.

Table 5.1: Approaches, Data Variants, and Methods (same as Table 1.1; reproduced here for convenience)

Time Interval	Approach	Data Variant 1	Data Variant 2
		Directly Obtained from Bus Passes	Bus Passes after Regression-based Transformation
Discrete	Arithmetic Average	Method 1	Method 2
Continuous	Integral-Benchmark interpolation	Method 3	Method 4
	Smoothed interpolation	Method 5	Method 6

Three metrics are considered to quantify the comparisons of the aggregated bus pass volume estimates to the corresponding road tube volumes estimates: *difference*, *absolute* difference, and *absolute relative error*. Metric values are determined separately for each Method by Case (implemented to address exceptionally low- and high-volume estimates on individual bus passes), estimation period (15- or 60-miutes), and Data Variant (Variant 1 or 2). Means, standard deviations, and medians are used to summarize the empirical distributions of the metrics, where the distributions are taken across the multiple time intervals corresponding to the estimation periods and the multiple road segments in the data base and contain hundreds of observations.

Based on the findings from the evaluations, there is no evidence of systematic over- or underestimation of period volumes for any Method. After considering comparisons by estimation period, Case, and Data Variant, the so-called continuous time approaches (i.e., the integral-benchmark and integrated smoothed interpolation approaches) perform consistently better than the so-called discrete time approach (i.e., arithmetic average).

Comparing the integral-benchmark interpolation approach to the smoothed interpolation approach reveals that there is no clear indication of which approach performs better, and the performance of approaches depends on the metrics used and whether the mean or median is used to summarize the distributions of the metrics, as well as the estimation period, Case, and Data Variant considered.

Unlike the finding that the integral Methods perform better than the arithmetic average Method, there is no clear indication of which of the integral Methods (integral-benchmark interpolation or smoothed interpolation) performs better, A few consistent "secondary" findings are also noted. As mentioned above, no systematic over- or underestimation of period volumes is found for any Method, estimation period, Case, or Data Variant. It is also found that lower means and medians of magnitude of relative errors (i.e., means and medians of *ARE*s) are obtained for 60-minute estimates than for 15-minute estimates for each Method, Case, and Data Variant. In addition, the magnitudes of errors, as represented by *absolute differences* and *ARE*s, obtained when using Data Variant 2 (i.e., transforming the individual bus pass volume estimates through a regression model) are systematically lower than the magnitudes of the errors obtained with Data Variant 1 (i.e., using the volumes directly obtained from individual bus passes).

5.2. Suggestion for Future Research

The results of the empirical investigations conducted in this this study motivate several dimensions for future research. The aggregation approaches in continuous time (integral-benchmark interpolation and integrated smoothed interpolation) outperformed the aggregation approach in discrete periods (arithmetic average). However, whether integral-benchmark interpolation performed better than integrated smoothed interpolation depended on the estimation period, Case, and the summary statistics considered. It is noted that only one smoothing method (moving median time window) with one set of parameter values (a window time of 30-minutes with step size of 1-minute) was implemented in this thesis. Investigation of other smoothing methods and when and why a specific implementation performs well would be warranted. For example, it was found that the smoothed interpolation approach implemented in this thesis performed differently, compared to the benchmark interpolation approach, when considering 15- and 60-minute estimation periods. It would be interesting to investigate the cause associated with this different performance. It would also be interesting to assess the performance when using window sizes other than 30-minutes and step sizes other than 1-minute. In addition to the moving median time window technique, different smoothing techniques can be explored to understand properties that might make them better or worse suited for aggregating volumes obtained from individual bus passes. For example, the moving median technique that determines the flow rate to assign to the midpoint of the time window as the median of all the volumes in the window could be replaced with a moving mean or a moving mean weighted by difference in time from the midpoint. As another example, rather than considering a window of time (e.g., a window of 30-minutes), a window of a number of consecutive flow rates (e.g., a window corresponding to flow rates obtained from five consecutive bus passes) could be considered, and these flow rates could smooth using either median or mean.

In this study, summary statistics obtained from each Method by segment are placed in an appendix. It was (casually) observed that different performance was seen on different segments. A closer look at the performance across segments would be of interest. If large differences are found, it would be interesting to see if differences among the segments could be associated with different properties of the segments (e.g., number of lanes, length of the segment, type of traffic control) or of the volumes or numbers of bus passes.

The Methods were investigated in terms of their ability to provide good estimations of 15- and 60-minute estimation periods. Other estimation periods – for example periods spanning the length of data collection (12-hours in this case) or across multiple days of data collection (which would require addition empirical data) – or other traffic measures derived from traffic volumes – for example, vehicle mile traveled – could be considered.

A regression model was estimated to transform the volumes directly obtained from the individual bus passes into Data Variant 2 before aggregation. Although not the focus of this thesis, it was found that each aggregation approach led to lower errors when using Data Variant 2 than when using Data Variant 1. It appears that the regression transformation leads to better volume estimates for the individual bus passes, even though the calibration data set and prediction transformation set are different and even though the regression specifications are rather arbitrarily chosen. It should be confirmed that the volumes for the individual bus passes are, indeed, better after transforming through the regression model compared to before transformation. If this finding is confirmed, it would be useful to investigate better regression specifications.

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Appendix A: Summary statistics of the metrics obtained from Methods 1 and 2, and regression summary from each specification

Summary statistics of the metrics by segment obtained when using Method 1 (arithmetic averaging of volume estimates directly obtained from bus passes) and Method 2 (arithmetic average of volume estimates from bus passes after regression-based transformation) in 15-and 60-minute estimation periods and for each Case

Estimated coefficients, t-statistics, p-values, adjusted R-squared values, and Tube Volumes calculated at Video Volumes of 7.5 and 150 vehicles/ln/15minutes obtained from each specification for all Cases using pooled 2018 and 2019 Data sets

Table A.1: Summary statistics of the metrics by segment obtained with Method 1 (arithmetic average of volume estimates from bus passes), for Case 1 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.435	17.327	-2.958	12.874	11.483	10.832	0.272	0.251	0.205
	1.2	0.570	22.036	-1.111	16.374	14.489	10.753	0.358	0.333	0.268
	4.1	1.320	11.466	0.105	8.477	7.726	6.279	0.135	0.112	0.097
	4.2	-3.288	11.520	0.889	9.384	7.326	7.747	0.170	0.101	0.172
	10.1	7.157	17.666	5.399	13.835	12.973	8.594	0.283	0.215	0.235
	10.2	3.310	16.080	3.625	12.376	10.558	9.671	0.364	0.388	0.277
	15.1	7.007	31.439	2.066	24.490	20.592	20.070	0.293	0.202	0.280
	15.2	29.079	39.633	23.266	35.770	33.522	24.240	0.372	0.305	0.281
	19.1	13.405	41.429	6.677	33.806	26.890	33.206	0.757	0.729	0.606
	19.2	-9.419	23.603	-6.586	19.681	15.809	16.646	0.363	0.279	0.302
2019	5.1	25.915	34.225	26.376	33.084	27.128	29.861	0.611	0.454	0.455
	5.2	1.971	18.998	3.531	15.003	11.579	12.626	0.288	0.262	0.253
	16.1	-11.205	24.317	-8.796	19.394	18.133	16.566	0.322	0.265	0.279
	16.2	29.570	53.640	8.169	35.092	50.097	11.394	0.541	0.630	0.346
	20.1	0.410	20.452	-2.565	14.523	14.207	10.766	0.382	0.328	0.330
	20.2	2.573	21.119	9.692	17.720	11.407	15.287	0.367	0.207	0.357
	24.1	15.612	23.970	11.614	21.683	18.463	18.381	0.340	0.309	0.308
	24.2	1.296	22.630	1.531	17.879	13.560	19.405	0.314	0.225	0.384

Table A.2: Summary statistics of the metrics by segment obtained with Method 1 (arithmetic average of volume estimates from bus passes), for Case 2 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.435	17.327	-2.958	12.874	11.483	10.832	0.272	0.251	0.205
	1.2	1.483	21.684	-0.959	15.958	14.494	9.895	0.339	0.319	0.267
	4.1	1.320	11.466	0.105	8.477	7.726	6.279	0.135	0.112	0.097
	4.2	-3.288	11.520	0.889	9.384	7.326	7.747	0.170	0.101	0.172
	10.1	7.157	17.666	5.399	13.835	12.973	8.594	0.283	0.215	0.235
	10.2	4.596	14.710	3.818	11.630	9.908	9.345	0.342	0.376	0.263
	15.1	0.878	23.550	-0.337	19.741	12.456	19.225	0.260	0.167	0.271
	15.2	4.701	19.314	5.437	15.041	12.616	12.499	0.231	0.216	0.154
	19.1	14.326	35.720	13.174	30.652	22.703	33.206	0.687	0.728	0.595
	19.2	-6.951	22.834	-3.300	17.620	15.860	12.297	0.319	0.266	0.217
2019	5.1	21.443	27.310	23.818	28.176	20.034	26.376	0.554	0.418	0.438
	5.2	1.971	18.998	3.531	15.003	11.579	12.626	0.288	0.262	0.253
	16.1	-11.205	24.317	-8.796	19.394	18.133	16.566	0.322	0.265	0.279
	16.2	6.896	18.171	6.047	13.750	13.557	8.915	0.281	0.217	0.222
	20.1	2.500	17.951	-0.152	12.937	12.516	8.915	0.354	0.319	0.306
	20.2	1.781	20.678	2.225	16.989	11.583	15.160	0.360	0.215	0.357
	24.1	15.467	28.368	14.341	24.965	20.206	19.917	0.393	0.331	0.353
	24.2	2.335	22.473	2.493	17.933	13.368	18.408	0.316	0.217	0.394

Table A.3: Summary statistics of the metrics by segment obtained with Method 1 (arithmetic average of volume estimates from bus passes), for Case 3 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.435	17.327	-2.958	12.874	11.483	10.832	0.272	0.251	0.205
	1.2	0.784	21.760	-1.111	16.159	14.329	10.753	0.350	0.322	0.268
	4.1	1.320	11.466	0.105	8.477	7.726	6.279	0.135	0.112	0.097
	4.2	-3.288	11.520	0.889	9.384	7.326	7.747	0.170	0.101	0.172
	10.1	7.157	17.666	5.399	13.835	12.973	8.594	0.283	0.215	0.235
	10.2	3.560	15.529	3.625	12.126	10.107	9.671	0.356	0.378	0.277
	15.1	2.980	24.206	2.066	20.462	12.879	19.984	0.260	0.167	0.270
	15.2	12.413	24.307	8.361	19.869	18.517	13.124	0.254	0.262	0.154
	19.1	13.079	38.469	6.677	31.890	24.647	27.696	0.712	0.704	0.606
	19.2	-9.003	23.185	-6.586	19.264	15.466	16.646	0.355	0.267	0.302
2019	5.1	24.118	30.145	26.376	31.073	22.662	29.861	0.582	0.421	0.455
	5.2	1.971	18.998	3.531	15.003	11.579	12.626	0.288	0.262	0.253
	16.1	-11.205	24.317	-8.796	19.394	18.133	16.566	0.322	0.265	0.279
	16.2	17.890	28.563	8.169	23.412	24.105	11.394	0.388	0.298	0.346
	20.1	0.706	19.821	-2.565	14.227	13.620	10.766	0.376	0.322	0.330
	20.2	2.364	20.896	9.692	17.510	11.281	15.287	0.365	0.209	0.357
	24.1	15.575	23.930	11.614	21.728	18.322	17.371	0.341	0.305	0.287
	24.2	1.530	22.456	1.531	17.645	13.613	19.405	0.307	0.221	0.384

Table A.4: Summary statistics of the metrics by segment obtained with Method 1 (arithmetic average of volume estimates from bus passes), for Case 1 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-5.664	36.534	-14.600	30.167	19.209	32.825	0.174	0.122	0.142
	1.2	12.305	48.856	19.124	41.965	24.825	31.757	0.206	0.116	0.207
	4.1	3.035	19.275	5.477	16.949	8.103	16.379	0.070	0.032	0.069
	4.2	-13.871	20.180	-6.171	16.556	17.814	7.702	0.068	0.050	0.058
	10.1	33.213	32.176	26.296	34.125	31.108	26.296	0.184	0.109	0.159
	10.2	2.703	43.198	18.708	35.444	22.186	28.139	0.222	0.135	0.169
	15.1	36.102	86.524	25.582	66.270	63.947	38.847	0.185	0.140	0.137
	15.2	99.430	115.087	71.189	114.698	98.229	83.970	0.287	0.171	0.322
	19.1	73.504	102.455	88.642	97.209	77.754	88.642	0.599	0.519	0.490
	19.2	-43.203	67.721	-37.172	61.516	49.765	40.674	0.267	0.127	0.214
2019	5.1	103.980	73.394	117.976	107.205	68.098	117.976	0.478	0.284	0.514
	5.2	4.523	38.199	0.725	27.721	25.226	20.285	0.133	0.127	0.074
	16.1	-51.685	99.858	-40.388	75.655	81.327	57.052	0.288	0.259	0.250
	16.2	143.249	207.981	27.213	153.391	199.866	33.860	0.554	0.643	0.200
	20.1	7.178	62.426	-9.508	42.403	44.451	27.600	0.230	0.149	0.170
	20.2	16.125	38.634	22.855	36.549	17.584	37.151	0.205	0.118	0.213
	24.1	58.635	65.633	77.118	78.923	35.185	77.118	0.312	0.152	0.321
	24.2	6.668	58.260	3.595	45.528	34.104	41.666	0.204	0.160	0.175

Table A.5: Summary statistics of the metrics by segment obtained with Method 1 (arithmetic average of volume estimates from bus passes), for Case 2 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-5.664	36.534	-14.600	30.167	19.209	32.825	0.174	0.122	0.142
	1.2	15.123	47.327	19.124	40.147	26.806	31.757	0.191	0.125	0.154
	4.1	3.035	19.275	5.477	16.949	8.103	16.379	0.070	0.032	0.069
	4.2	-13.871	20.180	-6.171	16.556	17.814	7.702	0.068	0.050	0.058
	10.1	33.213	32.176	26.296	34.125	31.108	26.296	0.184	0.109	0.159
	10.2	5.789	41.970	18.708	33.414	23.878	23.005	0.206	0.145	0.151
	15.1	10.019	50.341	18.463	40.187	29.464	33.522	0.127	0.102	0.107
	15.2	3.932	46.822	12.259	37.018	25.848	31.838	0.137	0.113	0.101
	19.1	78.074	94.497	88.642	95.482	74.875	88.642	0.648	0.620	0.490
	19.2	-32.198	66.818	-29.084	52.120	51.155	40.101	0.222	0.127	0.213
2019	5.1	87.073	61.943	68.555	90.298	56.636	68.555	0.423	0.268	0.335
	5.2	4.523	38.199	0.725	27.721	25.226	20.285	0.133	0.127	0.074
	16.1	-51.685	99.858	-40.388	75.655	81.327	57.052	0.288	0.259	0.250
	16.2	22.436	51.025	18.706	39.965	37.258	29.539	0.176	0.121	0.172
	20.1	15.493	56.256	9.074	35.737	44.988	26.678	0.198	0.153	0.154
	20.2	5.114	48.135	11.130	40.806	22.675	39.034	0.216	0.113	0.220
	24.1	56.948	71.175	77.118	83.132	32.112	79.041	0.323	0.138	0.321
	24.2	11.216	52.154	3.595	40.979	31.799	33.418	0.176	0.119	0.175

Table A.6: Summary statistics of the metrics by segment obtained with Method 1 (arithmetic average of volume estimates from bus passes), for Case 3 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-5.664	36.534	-14.600	30.167	19.209	32.825	0.174	0.122	0.142
	1.2	12.987	48.325	19.124	41.284	25.418	31.757	0.201	0.117	0.154
	4.1	3.035	19.275	5.477	16.949	8.103	16.379	0.070	0.032	0.069
	4.2	-13.871	20.180	-6.171	16.556	17.814	7.702	0.068	0.050	0.058
	10.1	33.213	32.176	26.296	34.125	31.108	26.296	0.184	0.109	0.159
	10.2	3.385	42.720	18.708	34.762	22.546	23.005	0.217	0.136	0.159
	15.1	17.382	56.365	25.582	47.550	31.973	38.847	0.146	0.103	0.114
	15.2	39.522	51.957	38.000	54.789	33.401	39.942	0.172	0.128	0.129
	19.1	69.505	94.267	80.926	89.119	73.919	80.926	0.559	0.523	0.353
	19.2	-41.566	66.822	-37.172	59.880	49.209	40.674	0.259	0.120	0.214
2019	5.1	98.944	67.272	117.976	102.170	61.742	117.976	0.460	0.270	0.486
	5.2	4.523	38.199	0.725	27.721	25.226	20.285	0.133	0.127	0.074
	16.1	-51.685	99.858	-40.388	75.655	81.327	57.052	0.288	0.259	0.250
	16.2	84.151	113.091	27.213	94.293	103.917	33.860	0.355	0.325	0.200
	20.1	8.405	61.119	-9.508	41.175	44.139	27.600	0.224	0.146	0.170
	20.2	14.249	38.007	16.516	34.673	18.585	26.887	0.200	0.124	0.213
	24.1	58.145	64.782	77.118	76.250	38.989	77.118	0.301	0.164	0.321
	24.2	7.759	56.489	3.595	44.437	32.942	41.666	0.198	0.145	0.175

Table A.7: Estimated coefficients, t-statistics, p-values, adjusted R-squared values, and Tube Volumes calculated at Video Volumes of 7.5 and 150 vehicles/ln/15minutes obtained from each specification for Case 1 using pooled 2018 and 2019 Data sets

Alpha	Coefficients		t-statistics	P-value	Adjusted R-squared	Calculated Tube Volume using Video Volume of 7.5 veh/ln/15-min	Calculated Tube Volume using Video Volume of 150 veh/ln/15-min
1	Intercept	31.39871	31.74	2e-16 ***	0.4786	34.3747	90.9187
	Slope	0.3968	29.81	2e-16 ***			
0.1	Intercept	32.7943	8.695	2e-16 ***	0.4781	34.4450	91.0157
	b1	0.4003	24.867	2e-16 ***			
	b2	-1.1049	-0.383	0.701			
0.2	Intercept	30.558	8.48	2e-16 ***	0.4781	34.2472	90.8498
	b1	0.3929	18.935	2e-16 ***			
	b2	0.4958	0.243	0.808			
0.3	Intercept	28.7083	8.494	2e-16 ***	0.4784	33.7265	90.6528
	b1	0.3771	13.915	2e-16 ***			
	b2	1.1965	0.832	0.405			
0.4	Intercept	27.33848	8.703	2e-16 ***	0.4790	33.0810	90.4779
	b1	0.35245	10.020	2e-16 ***			
	b2	1.38425	1.362	0.174			

Continued

Table A.7 Continued

Alpha	Coefficients		t-statistics	P-value	Adjusted R-squared	Calculated Tube Volume using Video Volume of 7.5 veh/ln/15-min	Calculated Tube Volume using Video Volume of 150 veh/ln/15-min
0.5	Intercept	26.39498	9.063	2e-16 ***	0.4798	32.4405	90.3266
	b1	0.31681	6.922	8.11e-12 ***			
	b2	1.33988	1.826	0.0681 .			
0.6	Intercept	25.77866	9.534	2e-16 ***	0.4807	31.8696	90.2090
	b1	0.26423	4.343	1.55e-05 ***			
	b2	1.22666	2.233	0.0258 *			
0.7	Intercept	25.39643	10.081	2e-16 ***	0.4816	31.3903	90.1285
	b1	0.17913	2.105	0.03558 *			
	b2	1.13487	2.589	0.00977 **			
0.8	Intercept	25.1763	10.677	2e-16 ***	0.4826	31.0028	90.0827
	b1	0.0133	0.100	0.9202			
	b2	1.1425	2.905	0.00376 **			
0.9	Intercept	25.067	11.302	2e-16 ***	0.4835	30.6978	90.0663
	b1	-0.4755	-1.735	0.08310 .			
	b2	1.500	3.186	0.00149 **			

Continued

Table A.7 Continued

Alpha	Coefficients		t-statistics	P-value	Adjusted R-squared	Calculated Tube Volume using Video Volume of 7.5 veh/ln/15-min	Calculated Tube Volume using Video Volume of 150 veh/ln/15-min
1.1	Intercept	25.0503	12.59	2e-16 ***	0.4852	30.2881	90.0903
	b1	1.4564	5.039	5.6e-07 ***			
	b2	-0.6197	-3.67	0.000256 ***			
1.2	Intercept	25.1025	13.234	2e-16 ***	0.4861	30.1624	90.1514
	b1	0.96834	6.547	9.51e-11 ***			
	b2	-0.19628	-3.880	0.000112 ***			
1.3	Intercept	25.17845	13.871	2e-16 ***	0.4869	30.0763	90.2182
	b1	0.80371	7.976	4.28e-15 ***			
	b2	-0.08232	-4.073	5.02e-05 ***			
1.4	Intercept	25.270399	14.495	2e-16 ***	0.4876	30.0227	90.3027
	b1	0.720086	9.33	2e-16 ***			
	b2	-0.038614	-4.251	2.33e-05 ***			
1.5	Intercept	25.372987	15.106	2e-16 ***	0.4884	29.9954	90.3995
	b1	0.668967	10.615	2e-16 ***			
	b2	-0.019225	-4.417	1.12e-05 ***			

Continued

Table A.7 Continued

Alpha	Coefficients		t-statistics	P-value	Adjusted R-squared	Calculated Tube Volume using Video Volume of 7.5 veh/ln/15-min	Calculated Tube Volume using Video Volume of 150 veh/ln/15-min
1.6	Intercept	25.482439	15.70	2e-16 ***	0.4891	29.9893	90.5087
	b1	0.634174	11.84	2e-16 ***			
	b2	-0.009927	-4.57	5.51e-06 ***			
1.7	Intercept	25.596081	16.276	2e-16 ***	0.4898	30.0004	90.6274
	b1	0.608764	12.991	2e-16 ***			
	b2	-0.005252	-4.713	2.8e-06 ***			
1.8	Intercept	25.712004	16.834	2e-16 ***	0.4904	30.0252	90.7580
	b1	0.5892583	14.085	2e-16 ***			
	b2	-0.0028261	-4.847	1.46e-06 ***			
1.9	Intercept	25.828836	17.373	2e-16 ***	0.4911	30.0609	90.8942
	b1	0.5737195	15.121	2e-16 ***			
	b2	-0.0015399	-4.971	7.87e-07 ***			
2	Intercept	25.9455885	17.893	2e-16 ***	0.4917	30.1053	91.0354
	b1	0.56098	16.099	2e-16 ***			
	b2	-0.00085	-5.088	4.35e-07 ***			

Continued

Table A.8: Estimated coefficients, t-statistics, p-values, adjusted R-squared values, and Tube Volumes calculated at Video Volumes of 7.5 and 150 vehicles/ln/15minutes obtained from each specification for Case 2 using pooled 2018 and 2019 Data sets

Alpha	Coefficients		t-statistics	P-value	Adjusted R-squared	Calculated Tube Volume using Video Volume of 7.5 veh/ln/15-min	Calculated Tube Volume using Video Volume of 150 veh/ln/15-min
1	Intercept	27.8039	24.19	2e-16 ***	0.4284	31.2744	97.2134
	Slope	0.46273	26.11	2e-16 ***			
0.1	Intercept	53.45683	2.064	0.0393 *	0.4284	33.6119	98.7383
	b1	0.51514	9.239	2e-16 ***			
	b2	-19.3820	-0.991	0.3217			
0.2	Intercept	40.2830	3.033	0.00249 **	0.4284	33.4266	98.7194
	b1	0.52014	8.203	7.97e-16 ***			
	b2	-7.18948	-0.943	0.34588			
0.3	Intercept	35.9064	3.952	8.34e-05 ***	0.4283	33.2589	98.7029
	b1	0.52669	7.184	1.41e-12 ***			
	b2	-3.60474	-0.899	0.369			
0.4	Intercept	33.7299	4.824	1.65e-06 ***	0.4283	33.1078	98.6938
	b1	0.5356	6.184	9.44e-10 ***			
	b2	-2.0721	-0.859	0.39			

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Table A.8 Continued

Alpha	Coefficients		t-statistics	P-value	Adjusted R-squared	Calculated Tube Volume using Video Volume of 7.5 veh/ln/15-min	Calculated Tube Volume using Video Volume of 150 veh/ln/15-min
0.5	Intercept	32.4338	5.649	2.16e-08 ***	0.4282	32.9714	98.6809
	b1	0.5482	5.206	2.39e-07 ***			
	b2	-1.305	-0.823	0.411			
0.6	Intercept	31.5782	6.431	2.05e-10 ***	0.4282	32.8485	98.6644
	b1	0.5673	4.251	2.35e-05 ***			
	b2	-0.8909	-0.791	0.429			
0.7	Intercept	30.9744	7.17	1.56e-12 ***	0.4282	32.7387	98.6564
	b1	0.5995	3.321	0.000931 ***			
	b2	-0.6667	-0.761	0.446629			
0.8	Intercept	30.5279	7.87	1.01e-14 ***	0.4281	32.6400	98.6429
	b1	0.6643	2.418	0.0158 *			
	b2	-0.5726	-0.735	0.4624			
0.9	Intercept	30.1864	8.532	2e-16 ***	0.4281	32.5520	98.6375
	b1	0.8598	1.541	0.124			
	b2	-0.6659	-0.712	0.477			

Continued

Table A.8 Continued

Alpha	Coefficients		t-statistics	P-value	Adjusted R-squared	Calculated Tube Volume using Video Volume of 7.5 veh/ln/15-min	Calculated Tube Volume using Video Volume of 150 veh/ln/15-min
1.1	Intercept	29.70332	9.753	2e-16 ***	0.4281	32.4039	98.6464
	b1	0.07506	0.13	0.896			
	b2	0.233	0.674	0.501			
1.2	Intercept	29.52836	10.316	2e-16 ***	0.4281	32.3414	98.6512
	b1	0.27057	0.925	0.355			
	b2	0.06984	0.658	0.511			
1.3	Intercept	29.38401	10.85	2e-16 ***	0.4281	32.2857	98.6574
	b1	0.33544	1.692	0.0911 .			
	b2	0.02811	0.644	0.5195			
1.4	Intercept	29.26362	11.357	2e-16 ***	0.4281	32.2362	98.6712
	b1	0.36766	2.431	0.0152 *			
	b2	0.01281	0.633	0.5269			
1.5	Intercept	29.162324	11.838	2e-16 ***	0.4280	32.1920	98.6825
	b1	0.386823	3.144	0.00172 **			
	b2	0.006258	0.624	0.53311			

Continued

Table A.8 Continued

Alpha	Coefficients		t-statistics	P-value	Adjusted R-squared	Calculated Tube Volume using Video Volume of 7.5 veh/ln/15-min	Calculated Tube Volume using Video Volume of 150 veh/ln/15-min
1.6	Intercept	29.076444	12.295	2e-16 ***	0.4280	32.1528	98.6982
	b1	0.39946	3.831	0.000136 ***			
	b2	0.0032	0.616	0.538199			
1.7	Intercept	29.003168	12.729	2e-16 ***	0.4280	32.1179	98.7165
	b1	0.408372	4.493	7.95e-06 ***			
	b2	0.00169	0.61	0.542			
1.8	Intercept	2.89E+01	13.143	2e-16 ***	0.4280	32.0869	98.7468
	b1	4.15E-01	5.129	3.56e-07 ***			
	b2	9.15E-04	0.605	0.545			
1.9	Intercept	2.89E+01	13.536	2e-16 ***	0.4280	32.0632	98.7703
	b1	4.20E-01	5.741	1.28e-08 ***			
	b2	5.05E-04	0.602	0.547			
2	Intercept	2.88E+01	13.911	2e-16 ***	0.4280	32.0359	98.8008
	b1	4.24E-01	6.331	3.84e-10 ***			
	b2	2.83E-04	0.60	0.549			

Continued

Table A.9: Estimated coefficients, t-statistics, p-values, adjusted R-squared values, and Tube Volumes calculated at Video Volumes of 7.5 and 150 vehicles/ln/15minutes obtained from each specification for Case 3 using pooled 2018 and 2019 Data sets

Alpha	Coefficients		t-statistics	P-value	Adjusted R-squared	Calculated Tube Volume using Video Volume of 7.5 veh/ln/15-min	Calculated Tube Volume using Video Volume of 150 veh/ln/15-min
1	Intercept	27.42862	25.64	2e-16 ***	0.4958	30.9802	98.4596
	Slope	0.47354	30.85	2e-16 ***			
0.1	Intercept	66.3251	3.203	0.0014 **	0.4971	34.5441	100.2770
	b1	0.54928	12.749	2e-16 ***			
	b2	-29.34917	-1.881	0.0603 .			
0.2	Intercept	47.1155	4.416	1.12e-05 ***	0.4971	34.4052	100.3155
	b1	0.5599	11.425	2e-16 ***			
	b2	-11.3011	-1.855	0.064 .			
0.3	Intercept	40.71878	5.549	3.72e-08 ***	0.4970	34.2714	100.3428
	b1	0.57352	10.11	2e-16 ***			
	b2	-5.87275	-1.831	0.0675 .			
0.4	Intercept	37.52497	6.603	6.66e-11 ***	0.4970	34.1440	100.3752
	b1	0.59178	8.814	2e-16 ***			
	b2	-3.49257	-2.441	0.0708 .			

Continued

Table A.9 Continued

Alpha	Coefficients		t-statistics	P-value	Adjusted R-squared	Calculated Tube Volume using Video Volume of 7.5 veh/ln/15-min	Calculated Tube Volume using Video Volume of 150 veh/ln/15-min
0.5	Intercept	35.6121	7.581	8.02e-14 ***	0.4970	34.0228	100.4073
	b1	0.61741	7.542	1.07e-13 ***			
	b2	-2.27119	-1.789	0.0739 .			
0.6	Intercept	34.3395	8.489	2e-16 ***	0.4969	33.9076	100.4345
	b1	0.6559	6.301	4.49e-10 ***			
	b2	-1.5974	-1.771	0.0768 .			
0.7	Intercept	33.4328	9.331	2e-16 ***	0.4969	33.7994	100.4781
	b1	0.7203	5.095	4.19e-07 ***			
	b2	-1.2289	-1.756	0.0795 .			
0.8	Intercept	32.7547	10.112	2e-16 ***	0.4969	33.6965	100.5067
	b1	0.8491	3.928	9.16e-05 ***			
	b2	-1.0826	-1.742	0.0819 .			
0.9	Intercept	32.2289	10.838	2e-16 ***	0.4969	33.6004	100.5476
	b1	1.2359	2.802	0.00517 **			
	b2	-1.2881	-1.73	0.08400 .			

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Table A.9 Continued

Alpha	Coefficients		t-statistics	P-value	Adjusted R-squared	Calculated Tube Volume using Video Volume of 7.5 veh/ln/15-min	Calculated Tube Volume using Video Volume of 150 veh/ln/15-min
1.1	Intercept	31.4682	12.139	2e-16 ***	0.4968	34.2275	122.2879
	b1	-0.3122	-0.679	0.4972			
	b2	0.556	1.71	0.0875 .			
1.2	Intercept	31.18474	12.723	2e-16 ***	0.4968	33.3442	100.6392
	b1	0.07455	0.317	0.7510			
	b2	0.14261	1.703	0.0889 .			
1.3	Intercept	30.94608	13.269	2e-16 ***	0.4968	33.2696	100.6718
	b1	0.20335	1.271	0.2040			
	b2	0.05816	1.697	0.0901 .			
1.4	Intercept	30.74256	13.779	2e-16 ***	0.4968	33.1998	100.7014
	b1	0.26767	2.183	0.0293 *			
	b2	0.02678	1.692	0.0293 *			
1.5	Intercept	30.567171	14.256	2e-16 ***	0.4968	33.1346	100.7353
	b1	0.306195	3.053	0.00233 **			
	b2	0.013194	1.688	0.09165 .			

Continued

Table A.9 Continued

Alpha	Coefficients		t-statistics	P-value	Adjusted R-squared	Calculated Tube Volume using Video Volume of 7.5 veh/ln/15-min	Calculated Tube Volume using Video Volume of 150 veh/ln/15-min
1.6	Intercept	30.414613	14.704	2e-16 ***	0.4968	33.0738	100.7696
	b1	0.33182	3.883	0.000209 ***			
	b2	0.006788	1.686	0.09212 .			
1.7	Intercept	30.280847	15.124	2e-16 ***	0.4968	33.0170	100.8031
	b1	0.350075	4.675	3.36e-06 ***			
	b2	0.003599	1.685	0.0924 .			
1.8	Intercept	30.162722	15.519	2e-16 ***	0.4968	32.9640	100.8361
	b1	0.363725	5.43	7.14e-08 ***			
	b2	0.001951	1.684	0.0925 .			
1.9	Intercept	3.01E+01	15.891	2e-16 ***	0.4968	32.9167	100.8735
	b1	3.74E-01	6.149	1.14e-09 ***			
	b2	1.08E-03	1.685	0.0924			
2	Intercept	29.960	16.241	2e-16 ***	0.4968	32.8641	100.8920
	b1	0.383	6.834	1.46e-11 ***			
	b2	0.0006	1.686	0.0922 .			

Continued

Table A.10: Summary statistics of the metrics by segment obtained with Method 2 (arithmetic average of regression-based transformations of individual bus pass volumes) using SPEC 1 for Case 1 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-0.986	15.915	3.472	11.306	11.082	7.707	0.212	0.154	0.200
	1.2	0.098	16.121	1.305	13.851	7.898	14.076	0.304	0.197	0.250
	4.1	-7.417	18.429	-2.645	13.364	14.598	7.955	0.188	0.124	0.175
	4.2	-2.790	18.765	1.507	15.112	11.243	13.240	0.320	0.259	0.314
	10.1	6.039	10.435	8.027	10.093	6.465	9.755	0.278	0.252	0.214
	10.2	10.720	8.720	11.120	11.831	7.079	11.506	0.369	0.304	0.295
	15.1	-19.527	12.372	-19.687	19.852	11.829	19.687	0.234	0.113	0.258
	15.2	-20.179	19.314	-15.965	22.032	17.100	15.965	0.217	0.110	0.205
	19.1	5.839	17.242	9.059	14.827	10.282	13.689	0.354	0.345	0.254
	19.2	-3.349	17.381	0.584	13.153	11.649	9.746	0.247	0.178	0.226
2019	5.1	12.026	16.270	10.345	15.811	12.499	12.379	0.300	0.251	0.217
	5.2	-0.695	12.987	0.713	9.695	8.530	7.699	0.185	0.196	0.147
	16.1	-11.839	17.207	-9.974	16.052	13.153	14.691	0.240	0.173	0.233
	16.2	14.823	21.240	9.449	17.193	19.314	10.172	0.337	0.280	0.250
	20.1	6.017	14.338	8.433	12.443	9.152	11.996	0.436	0.448	0.303
	20.2	1.934	17.827	5.665	13.358	11.759	11.245	0.271	0.190	0.221
	24.1	0.257	14.264	-1.104	11.299	8.470	9.848	0.177	0.146	0.144
	24.2	-2.434	12.852	-1.435	9.233	9.126	6.951	0.152	0.117	0.117

Table A.11: Summary statistics of the metrics by segment obtained with Method 2 (arithmetic average of regression-based transformations of individual bus pass volumes) using SPEC 20 for Case 1 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.063	14.748	0.495	10.186	10.580	6.616	0.188	0.154	0.162
	1.2	0.070	15.745	-1.166	13.412	7.921	11.769	0.294	0.208	0.251
	4.1	-5.899	17.217	-0.503	12.024	13.566	7.526	0.171	0.120	0.148
	4.2	-2.383	17.032	1.052	13.653	10.254	11.903	0.288	0.231	0.292
	10.1	6.427	9.936	8.203	10.097	6.043	9.633	0.273	0.230	0.223
	10.2	9.887	9.588	10.838	11.625	7.298	10.838	0.361	0.313	0.299
	15.1	-17.779	14.137	-17.606	18.517	13.131	17.606	0.215	0.121	0.231
	15.2	-21.956	24.462	-13.248	24.423	21.921	13.459	0.231	0.139	0.201
	19.1	5.687	18.622	8.289	15.502	11.503	15.642	0.363	0.375	0.267
	19.2	-4.033	17.922	0.828	14.150	11.487	10.769	0.266	0.170	0.251
2019	5.1	12.156	16.284	10.713	15.953	12.461	12.757	0.304	0.252	0.222
	5.2	-0.607	13.008	0.830	9.716	8.530	7.577	0.186	0.198	0.150
	16.1	-11.789	17.371	-10.132	16.126	13.227	14.577	0.242	0.175	0.235
	16.2	14.496	20.465	9.586	16.831	18.534	10.210	0.331	0.271	0.243
	20.1	5.881	14.330	8.067	12.340	9.191	11.586	0.429	0.440	0.290
	20.2	1.954	17.830	5.402	13.361	11.762	11.426	0.271	0.192	0.222
	24.1	0.428	14.331	-0.741	11.329	8.549	9.818	0.178	0.148	0.139
	24.2	-2.349	12.924	-1.532	9.328	9.106	7.321	0.154	0.117	0.125

Table A.12: Summary statistics of the metrics by segment obtained with Method 2 (arithmetic average of regression-based transformations of individual bus pass volumes) using SPEC 1 for Case 2 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.352	14.036	1.070	9.966	9.835	6.729	0.184	0.143	0.146
	1.2	-0.139	15.252	-0.760	13.172	7.341	11.671	0.290	0.196	0.245
	4.1	-6.351	16.202	-0.793	11.485	12.991	6.310	0.161	0.114	0.143
	4.2	-3.163	16.932	0.159	13.578	10.406	11.272	0.282	0.225	0.271
	10.1	5.917	9.627	7.293	9.479	6.036	8.696	0.257	0.224	0.200
	10.2	9.698	8.896	9.896	11.097	7.001	9.910	0.345	0.304	0.268
	15.1	-16.138	12.965	-15.895	17.090	11.645	15.895	0.211	0.127	0.218
	15.2	-10.883	13.518	-10.451	13.677	10.528	11.072	0.179	0.108	0.150
	19.1	6.313	19.912	9.056	16.941	11.853	16.486	0.402	0.409	0.304
	19.2	-3.805	17.733	0.247	13.199	12.249	9.059	0.243	0.178	0.202
2019	5.1	11.005	15.379	11.120	15.129	11.198	12.429	0.304	0.254	0.229
	5.2	-0.659	13.079	0.826	9.736	8.619	7.500	0.186	0.199	0.157
	16.1	-11.915	17.380	-10.653	16.259	13.186	14.779	0.245	0.175	0.236
	16.2	5.959	10.408	6.714	10.174	6.162	9.779	0.259	0.199	0.210
	20.1	6.895	12.946	8.993	11.852	8.491	11.524	0.421	0.429	0.265
	20.2	1.448	18.173	4.822	13.535	12.005	11.239	0.270	0.190	0.251
	24.1	0.919	17.911	0.197	13.388	11.690	10.505	0.212	0.190	0.149
	24.2	-1.800	13.545	0.043	9.776	9.390	8.048	0.166	0.141	0.126

Table A.13: Summary statistics of the metrics by segment obtained with Method 2 (arithmetic average of regression-based transformations of individual bus pass volumes) using SPEC 20 for Case 2 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.375	14.075	0.795	9.941	9.920	6.706	0.183	0.145	0.145
	1.2	-0.149	15.327	-0.558	13.233	7.384	11.786	0.291	0.198	0.250
	4.1	-6.272	16.275	-0.549	11.490	13.038	6.420	0.161	0.114	0.141
	4.2	-3.095	16.859	0.207	13.505	10.363	11.290	0.281	0.224	0.273
	10.1	5.947	9.647	7.460	9.542	5.996	8.839	0.258	0.223	0.204
	10.2	9.709	8.982	9.919	11.142	7.052	10.035	0.347	0.306	0.272
	15.1	-16.077	12.966	-16.162	17.019	11.666	16.162	0.209	0.126	0.215
	15.2	-10.816	13.649	-10.333	13.653	10.657	10.943	0.179	0.109	0.147
	19.1	6.309	19.953	9.077	16.976	11.869	16.452	0.403	0.408	0.308
	19.2	-3.769	17.787	0.171	13.245	12.264	9.222	0.244	0.179	0.206
2019	5.1	10.944	15.572	11.460	15.123	11.410	12.882	0.303	0.256	0.221
	5.2	-0.739	13.080	0.482	9.713	8.654	7.398	0.184	0.195	0.153
	16.1	-11.948	16.753	-10.412	15.973	12.764	15.161	0.240	0.169	0.226
	16.2	6.093	10.644	7.007	10.309	6.463	9.762	0.265	0.205	0.205
	20.1	7.264	13.180	8.976	12.229	8.616	12.546	0.440	0.454	0.279
	20.2	1.492	18.014	5.391	13.408	11.918	10.624	0.268	0.183	0.231
	24.1	1.155	17.806	0.355	13.481	11.437	11.085	0.213	0.188	0.147
	24.2	-1.857	13.453	0.199	9.722	9.326	7.847	0.164	0.137	0.121

Table A.14: Summary statistics of the metrics by segment obtained with Method 2 (arithmetic average of regression-based transformations of individual bus pass volumes) using SPEC 1 for Case 3 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.301	14.518	1.671	10.246	10.224	6.488	0.190	0.145	0.152
	1.2	-0.038	15.281	1.111	13.181	7.393	12.292	0.290	0.193	0.241
	4.1	-6.712	16.821	-1.006	11.982	13.494	6.658	0.168	0.117	0.153
	4.2	-3.110	17.444	0.779	14.012	10.644	11.779	0.293	0.234	0.282
	10.1	5.898	9.774	7.308	9.579	6.091	9.173	0.261	0.231	0.197
	10.2	9.721	8.807	10.214	10.976	7.120	10.333	0.342	0.304	0.271
	15.1	-18.284	13.687	-19.300	18.937	12.744	19.300	0.222	0.123	0.221
	15.2	-20.929	23.024	-13.085	23.522	20.282	13.525	0.227	0.127	0.221
	19.1	6.298	18.397	9.228	15.560	11.388	14.484	0.367	0.376	0.287
	19.2	-4.140	17.487	-0.013	13.411	11.765	9.428	0.249	0.170	0.209
2019	5.1	13.143	16.725	14.446	16.976	12.685	15.080	0.325	0.271	0.260
	5.2	-0.095	13.275	0.523	9.905	8.696	7.572	0.190	0.205	0.150
	16.1	-11.515	17.691	-11.124	16.281	13.197	14.404	0.248	0.181	0.224
	16.2	10.526	11.849	9.600	12.842	9.203	11.016	0.272	0.188	0.224
	20.1	5.660	14.056	6.558	11.911	9.208	11.274	0.403	0.410	0.252
	20.2	2.142	17.604	4.106	13.169	11.678	12.258	0.267	0.198	0.257
	24.1	2.558	14.979	1.203	12.101	8.942	10.016	0.193	0.165	0.156
	24.2	-1.558	13.569	-0.375	9.979	9.156	8.014	0.167	0.122	0.143

Table A.15: Summary statistics of the metrics by segment obtained with Method 2 (arithmetic average of regression-based transformations of individual bus pass volumes) using SPEC 20 for Case 3 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.266	14.495	0.656	10.096	10.340	6.912	0.186	0.149	0.153
	1.2	0.000	15.425	1.786	13.220	7.616	12.001	0.290	0.200	0.256
	4.1	-6.282	16.861	-0.162	11.841	13.459	6.755	0.167	0.118	0.144
	4.2	-2.825	17.103	0.640	13.708	10.412	11.803	0.287	0.229	0.285
	10.1	6.100	9.797	7.589	9.775	6.012	9.049	0.265	0.228	0.210
	10.2	9.713	9.121	10.346	11.108	7.290	10.506	0.345	0.310	0.282
	15.1	-18.007	14.105	-18.451	18.724	13.113	18.451	0.218	0.123	0.210
	15.2	-21.572	24.057	-13.373	24.168	21.363	13.527	0.230	0.134	0.213
	19.1	6.143	18.553	7.723	15.555	11.559	15.446	0.366	0.378	0.297
	19.2	-4.118	17.672	0.329	13.696	11.696	9.518	0.255	0.170	0.219
2019	5.1	12.825	17.279	13.115	16.837	13.268	14.245	0.318	0.268	0.226
	5.2	-0.647	13.175	0.319	9.737	8.761	7.351	0.184	0.194	0.147
	16.1	-11.882	16.230	-10.467	15.765	12.286	15.481	0.238	0.164	0.213
	16.2	11.569	13.676	10.481	14.088	10.978	11.039	0.295	0.205	0.251
	20.1	6.535	14.296	8.867	12.670	9.130	13.063	0.447	0.467	0.299
	20.2	2.045	17.289	5.576	12.887	11.511	10.110	0.262	0.179	0.216
	24.1	2.383	14.917	-0.353	11.990	8.943	10.064	0.191	0.162	0.156
	24.2	-2.006	13.230	-0.025	9.534	9.239	8.232	0.155	0.120	0.134

Table A.16: Summary statistics of the metrics by segment obtained with Method 2 (arithmetic average of regression-based transformations of individual bus pass volumes) using SPEC 1 for Case 1 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.757	55.773	23.239	39.632	37.232	29.409	0.184	0.109	0.183
	1.2	-3.317	53.834	-12.962	44.955	26.214	38.827	0.232	0.154	0.204
	4.1	-31.090	66.465	-17.822	44.493	57.474	22.349	0.143	0.124	0.111
	4.2	-11.358	68.510	-0.847	54.234	40.000	54.583	0.272	0.188	0.229
	10.1	27.253	32.913	33.033	35.905	21.958	37.699	0.237	0.186	0.203
	10.2	29.972	39.687	38.091	43.724	21.452	45.875	0.283	0.153	0.259
	15.1	-79.598	40.557	-70.992	79.598	40.557	70.992	0.234	0.073	0.229
	15.2	-85.186	67.147	-66.836	86.569	65.172	66.836	0.212	0.109	0.245
	19.1	30.595	59.680	32.480	56.186	33.449	44.906	0.362	0.337	0.232
	19.2	-17.382	70.972	11.563	45.659	55.464	22.852	0.183	0.147	0.122
2019	5.1	46.560	30.808	50.043	46.560	30.808	50.043	0.219	0.166	0.220
	5.2	-6.989	37.974	1.921	29.091	23.767	24.597	0.118	0.077	0.135
	16.1	-51.372	66.035	-36.979	61.336	55.904	36.979	0.215	0.160	0.175
	16.2	68.367	76.995	47.312	76.129	68.517	47.312	0.332	0.208	0.263
	20.1	20.346	48.836	26.472	43.315	27.865	42.742	0.362	0.408	0.215
	20.2	9.355	54.015	21.230	42.438	32.138	33.309	0.212	0.144	0.212
	24.1	0.720	39.116	-3.979	32.583	19.046	33.586	0.135	0.098	0.131
	24.2	-6.262	35.573	-2.998	0.082	0.091	0.039	20.883	28.789	11.309

Table A.17: Summary statistics of the metrics by segment obtained with Method 2 (arithmetic average of regression-based transformations of individual bus pass volumes) using SPEC 20 for Case 1 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-2.347	48.908	19.326	33.330	34.295	21.026	0.149	0.096	0.142
	1.2	-1.844	48.225	-11.914	40.076	23.721	36.763	0.208	0.149	0.158
	4.1	-25.258	61.563	-8.517	39.229	52.905	20.488	0.126	0.117	0.087
	4.2	-9.714	60.904	2.138	47.691	36.212	45.909	0.238	0.166	0.192
	10.1	28.691	29.557	33.293	35.083	20.620	33.293	0.233	0.178	0.204
	10.2	26.651	38.429	38.086	42.051	17.692	40.622	0.270	0.129	0.242
	15.1	-73.141	51.644	-61.765	73.141	51.644	61.765	0.208	0.095	0.197
	15.2	-90.658	88.534	-68.696	94.737	83.705	68.696	0.224	0.135	0.226
	19.1	30.620	61.277	32.593	54.605	38.812	48.995	0.355	0.366	0.271
	19.2	-20.523	70.528	4.701	46.128	55.650	20.577	0.184	0.150	0.133
2019	5.1	47.189	31.075	51.534	47.189	31.075	51.534	0.222	0.167	0.227
	5.2	-6.666	37.635	2.558	28.853	23.442	24.468	0.117	0.076	0.133
	16.1	-51.300	66.929	-37.432	61.674	56.483	37.432	0.217	0.162	0.180
	16.2	66.609	74.372	46.991	74.237	65.949	46.991	0.325	0.201	0.262
	20.1	19.921	48.255	27.229	42.715	27.532	42.300	0.355	0.397	0.212
	20.2	9.428	53.780	22.119	42.271	32.009	33.168	0.210	0.145	0.211
	24.1	1.308	39.474	-2.575	32.891	19.238	33.493	0.137	0.100	0.127
	24.2	-5.948	35.530	-2.467	20.975	28.590	12.426	0.083	0.091	0.042

Table A.18: Summary statistics of the metrics by segment obtained with Method 2 (arithmetic average of regression-based transformations of individual bus pass volumes) using SPEC 1 for Case 2 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-3.548	46.863	16.161	31.602	33.338	19.703	0.140	0.094	0.133
	1.2	-0.968	45.766	-17.073	39.462	19.560	34.289	0.211	0.150	0.167
	4.1	-26.956	56.457	-12.120	37.370	49.503	19.994	0.119	0.109	0.109
	4.2	-12.933	60.548	-2.676	47.868	36.464	43.987	0.234	0.158	0.204
	10.1	26.998	25.927	28.513	31.430	19.681	28.513	0.212	0.170	0.175
	10.2	25.744	38.591	37.660	41.283	18.535	46.393	0.266	0.131	0.246
	15.1	-72.921	46.896	-61.295	72.921	46.896	61.295	0.210	0.093	0.204
	15.2	-67.383	59.716	-67.935	71.012	54.781	67.935	0.202	0.113	0.213
	19.1	37.878	70.648	42.045	65.936	42.273	47.544	0.439	0.401	0.263
	19.2	-18.299	69.320	2.788	40.841	57.782	18.146	0.157	0.158	0.090
2019	5.1	40.522	36.994	38.554	42.911	33.901	38.554	0.211	0.181	0.189
	5.2	-6.797	37.051	2.410	28.421	23.131	23.288	0.114	0.076	0.125
	16.1	-51.834	67.280	-38.838	62.268	56.719	38.838	0.220	0.164	0.180
	16.2	13.866	35.460	28.085	32.429	17.395	38.547	0.190	0.136	0.180
	20.1	23.486	42.623	30.811	39.718	26.232	33.798	0.340	0.386	0.183
	20.2	3.574	62.900	22.914	46.222	40.253	31.639	0.217	0.148	0.210
	24.1	2.601	47.866	11.385	38.516	25.821	34.571	0.158	0.112	0.156
	24.2	-3.633	35.307	-2.240	20.592	28.190	8.809	0.081	0.089	0.044

Table A.19: Summary statistics of the metrics by segment obtained with Method 2 (arithmetic average of regression-based transformations of individual bus pass volumes) using SPEC 20 for Case 2 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-3.649	46.810	16.159	31.413	33.472	19.142	0.139	0.094	0.129
	1.2	-1.029	46.026	-16.556	39.538	20.000	34.972	0.211	0.151	0.164
	4.1	-26.667	56.897	-11.398	37.379	49.826	20.568	0.119	0.110	0.106
	4.2	-12.648	60.213	-2.108	47.464	36.378	43.578	0.232	0.157	0.201
	10.1	27.066	26.232	28.939	31.734	19.652	28.939	0.213	0.170	0.178
	10.2	25.700	38.591	37.955	41.409	18.156	46.215	0.266	0.129	0.245
	15.1	-73.086	48.150	-61.145	73.086	48.150	61.145	0.210	0.095	0.202
	15.2	-67.342	60.540	-68.529	71.141	55.435	68.529	0.202	0.115	0.213
	19.1	37.962	70.835	42.868	66.209	42.200	47.891	0.441	0.400	0.265
	19.2	-18.203	69.397	2.469	40.792	57.880	17.855	0.157	0.159	0.090
2019	5.1	40.275	37.153	37.150	42.225	34.690	37.150	0.208	0.182	0.182
	5.2	-6.917	37.533	1.063	28.884	23.316	23.519	0.117	0.076	0.129
	16.1	-51.517	64.257	-37.664	60.927	54.452	37.664	0.215	0.156	0.168
	16.2	15.039	36.450	30.553	34.228	16.679	38.609	0.198	0.136	0.193
	20.1	24.975	44.773	29.758	41.989	27.412	40.894	0.360	0.416	0.177
	20.2	3.668	62.400	21.852	46.094	39.642	31.456	0.217	0.144	0.236
	24.1	3.509	46.776	13.578	37.212	26.045	33.797	0.154	0.112	0.152
	24.2	-3.932	35.380	-2.645	21.013	27.983	10.117	0.082	0.086	0.046

Table A.20: Summary statistics of the metrics by segment obtained with Method 2 (arithmetic average of regression-based transformations of individual bus pass volumes) using SPEC 1 for Case 3 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-3.251	49.341	17.975	33.808	34.480	22.758	0.152	0.097	0.151
	1.2	-2.399	47.281	-16.110	40.519	20.878	34.392	0.212	0.142	0.178
	4.1	-28.360	59.311	-13.974	39.405	51.909	20.371	0.126	0.114	0.110
	4.2	-12.698	62.828	-2.368	49.715	37.497	46.811	0.245	0.166	0.212
	10.1	26.854	27.838	29.593	32.576	20.021	29.593	0.218	0.173	0.182
	10.2	26.126	38.552	37.567	41.115	19.433	39.273	0.264	0.135	0.251
	15.1	-74.497	46.871	-64.295	74.497	46.871	64.295	0.215	0.089	0.213
	15.2	-87.149	80.985	-66.901	89.624	77.957	66.901	0.212	0.129	0.227
	19.1	32.872	61.687	31.489	57.432	36.954	45.553	0.371	0.354	0.252
	19.2	-20.726	69.920	5.090	44.553	56.372	20.832	0.175	0.150	0.126
2019	5.1	52.487	34.088	60.053	52.625	33.854	60.053	0.248	0.180	0.219
	5.2	-4.475	35.869	0.660	27.631	21.651	23.426	0.112	0.076	0.136
	16.1	-50.462	69.270	-39.501	62.323	57.679	39.555	0.222	0.169	0.176
	16.2	46.411	43.729	45.536	53.826	33.084	45.536	0.253	0.118	0.255
	20.1	20.198	45.813	31.227	41.275	25.953	46.503	0.332	0.358	0.197
	20.2	10.176	50.384	23.409	40.095	29.733	31.251	0.200	0.145	0.185
	24.1	9.257	39.663	10.702	33.597	20.636	36.703	0.144	0.112	0.119
	24.2	-2.945	35.081	-4.239	21.155	27.349	8.112	0.086	0.090	0.043

Table A.21: Summary statistics of the metrics by segment obtained with Method 2 (arithmetic average of regression-based transformations of individual bus pass volumes) using SPEC 20 for Case 3 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-3.157	48.492	17.926	32.888	34.243	20.829	0.146	0.096	0.141
	1.2	-2.081	47.249	-14.497	40.038	21.770	36.007	0.210	0.145	0.166
	4.1	-26.725	59.749	-11.025	38.748	52.001	22.302	0.124	0.115	0.100
	4.2	-11.525	61.280	-0.436	48.055	36.894	45.362	0.237	0.164	0.201
	10.1	27.534	28.206	30.924	33.388	20.081	30.924	0.223	0.174	0.190
	10.2	25.958	38.400	36.827	41.399	18.179	38.857	0.266	0.129	0.245
	15.1	-73.745	50.233	-62.608	73.745	50.233	62.608	0.211	0.094	0.207
	15.2	-89.369	86.011	-68.367	92.446	82.356	68.367	0.218	0.136	0.224
	19.1	32.236	61.618	31.758	56.520	37.762	47.506	0.367	0.359	0.262
	19.2	-20.780	70.035	4.031	44.879	56.252	19.748	0.177	0.151	0.128
2019	5.1	50.797	34.476	52.061	51.110	33.963	52.061	0.238	0.174	0.229
	5.2	-6.297	37.451	0.563	28.976	22.861	23.479	0.118	0.076	0.130
	16.1	-50.878	61.920	-37.066	59.733	52.504	37.066	0.211	0.150	0.165
	16.2	52.721	52.019	47.160	61.424	40.165	47.160	0.283	0.133	0.257
	20.1	23.436	50.280	28.466	45.803	28.716	53.997	0.378	0.432	0.201
	20.2	10.053	49.078	22.486	39.299	28.654	31.390	0.199	0.135	0.196
	24.1	9.053	36.539	7.141	30.118	20.675	34.733	0.129	0.109	0.104
	24.2	-4.748	34.973	-4.909	20.875	27.724	10.394	0.080	0.085	0.038

Appendix B: Summary statistics of the metrics obtained from Methods 3, 4, 5, and 6

Summary statistics of the metrics by segment obtained when using Method 3 (integral-benchmark approach with flow rates derived directly obtained from individual bus passes), Method 4 – SPEC 20 (approach with flow rates from individual bus passes after regression-based transformation), Method 5 (smoothed curve approach with flow rates derived directly from individual bus passes), and Method 6 – SPEC 20 (smoothed curve approach with flow rates from individual bus passes after regression-based transformation) in 15-and 60-minute estimation periods and for each Case.

Table B.1: Summary statistics of the metrics by segment obtained with Method 3 (integral-benchmark approach applied to volume estimates directly obtained from individual bus passes), for Case 1 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.496	12.263	-3.431	9.317	7.963	6.756	0.206	0.185	0.132
	1.2	-0.484	16.363	-0.069	12.731	10.058	8.533	0.269	0.225	0.199
	4.1	1.348	9.448	1.147	7.163	6.211	5.948	0.118	0.099	0.091
	4.2	-3.751	10.166	-0.385	8.467	6.663	6.694	0.160	0.098	0.148
	10.1	8.701	13.988	7.552	12.264	10.912	8.568	0.262	0.205	0.228
	10.2	4.022	12.546	4.243	9.956	8.461	8.760	0.290	0.309	0.247
	15.1	7.934	26.237	3.309	20.411	18.053	17.359	0.243	0.181	0.211
	15.2	27.510	33.490	15.034	30.311	30.898	18.562	0.306	0.270	0.227
	19.1	16.124	33.860	15.583	28.785	23.667	20.928	0.662	0.586	0.529
	19.2	-9.845	18.983	-8.617	16.899	12.896	15.800	0.309	0.217	0.285
2019	5.1	26.550	21.599	27.659	27.892	19.781	27.659	0.508	0.335	0.477
	5.2	0.923	15.428	0.590	12.542	8.807	10.650	0.232	0.169	0.205
	16.1	-12.038	23.057	-9.648	19.262	17.168	14.401	0.317	0.255	0.240
	16.2	31.907	50.672	13.196	35.061	48.479	13.443	0.547	0.622	0.292
	20.1	0.178	18.900	-0.212	13.890	12.613	9.760	0.350	0.296	0.319
	20.2	2.392	17.663	2.590	14.565	9.992	12.266	0.295	0.181	0.270
	24.1	20.378	20.021	20.529	23.443	16.194	20.529	0.372	0.272	0.301
	24.2	1.000	17.692	1.735	13.397	11.348	10.904	0.233	0.195	0.214

Table B.2: Summary statistics of the metrics by segment obtained with Method 3 (integral-benchmark approach applied to volume estimates directly obtained from individual bus passes), for Case 2 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.501	12.262	-3.439	9.317	7.962	6.761	0.206	0.185	0.132
	1.2	0.388	16.036	0.205	12.212	10.180	8.072	0.248	0.213	0.188
	4.1	1.341	9.447	1.139	7.163	6.210	5.942	0.118	0.099	0.091
	4.2	-3.756	10.167	-0.391	8.468	6.666	6.689	0.160	0.098	0.148
	10.1	8.695	13.986	7.546	12.259	10.909	8.567	0.262	0.205	0.228
	10.2	5.141	11.268	4.940	9.328	8.014	8.134	0.271	0.301	0.246
	15.1	2.412	19.142	-0.181	15.882	10.646	14.702	0.210	0.150	0.181
	15.2	4.982	13.397	6.272	10.253	9.765	8.044	0.160	0.181	0.115
	19.1	15.700	26.733	19.508	26.120	16.229	20.917	0.615	0.536	0.523
	19.2	-7.319	18.043	-2.737	14.571	12.734	11.155	0.261	0.205	0.245
2019	5.1	22.478	18.827	25.907	23.903	16.921	25.907	0.473	0.343	0.471
	5.2	0.916	15.427	0.580	12.540	8.807	10.641	0.232	0.169	0.205
	16.1	-12.044	23.055	-9.656	19.264	17.167	14.399	0.317	0.255	0.240
	16.2	8.109	16.731	4.555	12.729	13.429	10.216	0.262	0.232	0.186
	20.1	2.592	15.986	0.060	11.927	10.781	9.237	0.318	0.291	0.261
	20.2	1.190	17.516	0.013	14.309	9.892	12.268	0.293	0.183	0.297
	24.1	16.778	22.221	17.231	22.784	15.765	18.768	0.363	0.271	0.243
	24.2	1.634	17.029	1.728	13.049	10.817	10.908	0.223	0.163	0.219

Table B.3: Summary statistics of the metrics by segment obtained with Method 3 (integral-benchmark approach applied to volume estimates directly obtained from individual bus passes), for Case 3 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.496	12.263	-3.431	9.317	7.963	6.756	0.206	0.185	0.132
	1.2	-0.470	16.347	-0.069	12.716	10.049	8.533	0.269	0.224	0.199
	4.1	1.348	9.448	1.147	7.163	6.211	5.948	0.118	0.099	0.091
	4.2	-3.751	10.166	-0.385	8.467	6.663	6.694	0.160	0.098	0.148
	10.1	8.701	13.988	7.552	12.264	10.912	8.568	0.262	0.205	0.228
	10.2	4.036	12.513	4.243	9.943	8.436	8.763	0.290	0.309	0.247
	15.1	4.382	20.010	3.309	16.859	11.347	16.809	0.214	0.151	0.174
	15.2	11.629	18.159	9.000	15.196	15.204	9.373	0.193	0.220	0.115
	19.1	14.808	31.088	15.583	27.596	20.180	20.928	0.642	0.549	0.529
	19.2	-9.816	18.954	-8.599	16.870	12.870	15.800	0.309	0.216	0.285
2019	5.1	24.740	19.520	27.345	26.083	17.629	27.345	0.483	0.323	0.471
	5.2	0.923	15.428	0.590	12.542	8.807	10.650	0.232	0.169	0.205
	16.1	-12.038	23.057	-9.648	19.262	17.168	14.401	0.317	0.255	0.240
	16.2	19.202	25.173	13.196	22.353	22.336	13.443	0.378	0.291	0.292
	20.1	0.199	18.860	-0.212	13.869	12.577	9.760	0.350	0.295	0.319
	20.2	2.174	17.446	2.590	14.383	9.830	12.266	0.293	0.181	0.270
	24.1	19.971	19.463	20.529	23.035	15.581	20.529	0.367	0.269	0.301
	24.2	1.020	17.667	1.735	13.386	11.324	10.904	0.232	0.193	0.214

Table B.4: Summary statistics of the metrics by segment obtained with Method 3 (integral-benchmark approach applied to volume estimates directly obtained from individual bus passes), for Case 1 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-12.482	29.626	-7.650	24.866	18.387	25.816	0.162	0.134	0.159
	1.2	-18.093	32.511	-24.009	31.984	15.665	31.822	0.173	0.082	0.178
	4.1	0.767	19.474	0.525	14.588	11.473	12.327	0.067	0.048	0.068
	4.2	-19.948	16.506	-12.230	19.948	16.506	12.230	0.088	0.049	0.077
	10.1	20.654	12.892	24.963	21.074	12.072	24.963	0.127	0.078	0.161
	10.2	13.797	49.335	24.914	40.132	27.874	26.842	0.259	0.176	0.197
	15.1	9.871	50.734	10.877	40.731	27.425	43.829	0.138	0.094	0.144
	15.2	68.805	82.148	64.262	84.525	62.739	64.262	0.241	0.133	0.265
	19.1	53.246	65.567	88.345	67.722	47.498	88.345	0.384	0.264	0.488
	19.2	-39.961	26.240	-46.756	41.592	23.094	46.756	0.223	0.114	0.238
2019	5.1	86.859	39.806	69.635	86.859	39.806	69.635	0.437	0.208	0.382
	5.2	11.801	32.106	13.576	27.228	18.120	29.859	0.123	0.073	0.110
	16.1	-35.940	69.775	-49.454	68.110	31.037	68.402	0.300	0.136	0.340
	16.2	53.017	76.426	32.459	64.383	65.453	33.848	0.266	0.213	0.192
	20.1	-9.912	34.256	-17.821	30.901	13.195	28.960	0.199	0.115	0.149
	20.2	14.979	37.331	14.202	33.716	18.142	33.561	0.201	0.108	0.221
	24.1	68.560	50.545	72.987	74.283	40.015	72.987	0.339	0.209	0.324
	24.2	18.792	44.303	24.415	39.719	23.113	36.452	0.195	0.134	0.174

Table B.5: Summary statistics of the metrics by segment obtained with Method 3 (integral-benchmark approach applied to volume estimates directly obtained from individual bus passes), for Case 2 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-12.482	29.626	-7.650	24.866	18.387	25.816	0.162	0.134	0.159
	1.2	-14.434	33.153	-24.009	29.064	18.886	31.822	0.149	0.098	0.152
	4.1	0.767	19.474	0.525	14.588	11.473	12.327	0.067	0.048	0.068
	4.2	-19.948	16.506	-12.230	19.948	16.506	12.230	0.088	0.049	0.077
	10.1	20.654	12.892	24.963	21.074	12.072	24.963	0.127	0.078	0.161
	10.2	13.797	49.335	24.914	40.132	27.874	26.842	0.259	0.176	0.197
	15.1	9.871	50.734	10.877	40.731	27.425	43.829	0.138	0.094	0.144
	15.2	-3.333	47.308	-3.391	35.771	26.755	43.534	0.130	0.110	0.123
	19.1	83.761	69.896	96.180	88.029	63.483	96.180	0.546	0.463	0.531
	19.2	-21.677	19.329	-19.246	23.308	16.971	19.246	0.128	0.095	0.104
2019	5.1	91.615	35.290	69.635	91.615	35.290	69.635	0.461	0.185	0.382
	5.2	11.801	32.106	13.576	27.228	18.120	29.859	0.123	0.073	0.110
	16.1	-35.940	69.775	-49.454	68.110	31.037	68.402	0.300	0.136	0.340
	16.2	10.037	36.289	21.322	32.172	15.064	33.848	0.152	0.077	0.145
	20.1	-6.999	31.059	-17.821	27.988	10.389	28.960	0.178	0.091	0.149
	20.2	14.979	37.331	14.202	33.716	18.142	33.561	0.201	0.108	0.221
	24.1	68.560	50.545	72.987	74.283	40.015	72.987	0.339	0.209	0.324
	24.2	21.630	37.546	24.415	36.881	19.199	36.452	0.177	0.095	0.174

Table B.6: Summary statistics of the metrics by segment obtained with Method 3 (integral-benchmark approach applied to volume estimates directly obtained from individual bus passes), for Case 3 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-12.482	29.626	-7.650	24.866	18.387	25.816	0.162	0.134	0.159
	1.2	-18.022	32.500	-24.009	31.913	15.716	31.822	0.173	0.082	0.174
	4.1	0.767	19.474	0.525	14.588	11.473	12.327	0.067	0.048	0.068
	4.2	-19.948	16.506	-12.230	19.948	16.506	12.230	0.088	0.049	0.077
	10.1	20.654	12.892	24.963	21.074	12.072	24.963	0.127	0.078	0.161
	10.2	13.797	49.335	24.914	40.132	27.874	26.842	0.259	0.176	0.197
	15.1	9.871	50.734	10.877	40.731	27.425	43.829	0.138	0.094	0.144
	15.2	40.236	54.841	37.983	55.956	35.260	55.021	0.184	0.128	0.179
	19.1	52.003	63.619	88.421	66.094	45.891	88.421	0.376	0.260	0.489
	19.2	-39.765	26.032	-46.652	41.396	22.875	46.652	0.222	0.113	0.238
2019	5.1	86.859	39.806	69.635	86.859	39.806	69.635	0.437	0.208	0.382
	5.2	11.801	32.106	13.576	27.228	18.120	29.859	0.123	0.073	0.110
	16.1	-35.940	69.775	-49.454	68.110	31.037	68.402	0.300	0.136	0.340
	16.2	53.017	76.426	32.459	64.383	65.453	33.848	0.266	0.213	0.192
	20.1	-9.912	34.256	-17.821	30.901	13.195	28.960	0.199	0.115	0.149
	20.2	14.979	37.331	14.202	33.716	18.142	33.561	0.201	0.108	0.221
	24.1	68.560	50.545	72.987	74.283	40.015	72.987	0.339	0.209	0.324
	24.2	18.792	44.303	24.415	39.719	23.113	36.452	0.195	0.134	0.174

Table B.7: Summary statistics of the metrics by segment obtained with Method 4 (integral-benchmark aggregation of regression-based transformation) using SPEC 20, for Case 1 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.156	14.175	2.996	9.480	10.482	6.199	0.171	0.128	0.167
	1.2	-0.288	14.875	1.139	12.903	7.068	12.069	0.280	0.175	0.259
	4.1	-5.920	17.053	-1.876	12.153	13.247	7.637	0.174	0.115	0.156
	4.2	-2.575	16.929	0.357	13.474	10.373	12.250	0.284	0.233	0.293
	10.1	6.982	9.441	8.990	9.960	6.115	10.092	0.275	0.252	0.207
	10.2	10.181	8.593	11.561	11.320	6.964	11.878	0.352	0.296	0.318
	15.1	-17.391	13.584	-16.544	18.175	12.487	16.544	0.210	0.108	0.212
	15.2	-22.189	24.562	-14.785	24.510	22.173	15.018	0.229	0.139	0.182
	19.1	6.578	16.827	10.630	14.835	10.041	12.834	0.357	0.337	0.282
	19.2	-4.226	16.941	-2.816	13.240	11.184	10.444	0.247	0.163	0.220
2019	5.1	12.457	9.797	11.062	13.212	8.719	11.062	0.261	0.191	0.195
	5.2	-1.135	12.061	0.321	8.747	8.266	6.369	0.162	0.160	0.127
	16.1	-12.216	17.334	-9.918	16.128	13.576	15.090	0.239	0.177	0.245
	16.2	15.590	18.935	11.145	17.125	17.516	11.145	0.348	0.294	0.303
	20.1	5.758	14.684	7.667	12.298	9.714	11.250	0.430	0.438	0.306
	20.2	1.853	17.015	6.065	11.974	12.069	8.046	0.245	0.192	0.205
	24.1	2.810	13.651	3.051	11.216	8.040	9.023	0.183	0.155	0.142
	24.2	-2.499	11.175	-1.323	7.900	8.179	4.854	0.131	0.112	0.087

Table B.8: Summary statistics of the metrics by segment obtained with Method 4 (integral-benchmark aggregation of regression-based transformation), using SPEC 20, for Case 2 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.417	13.405	2.300	9.140	9.790	5.635	0.165	0.120	0.149
	1.2	-0.574	14.466	-0.677	12.909	6.157	12.479	0.281	0.163	0.258
	4.1	-6.272	16.017	-3.046	11.680	12.536	6.907	0.166	0.106	0.140
	4.2	-3.285	16.739	-0.546	13.425	10.335	12.038	0.278	0.224	0.294
	10.1	6.544	9.038	7.972	9.321	6.042	9.180	0.259	0.244	0.198
	10.2	9.922	8.264	10.886	11.020	6.668	10.952	0.343	0.287	0.298
	15.1	-15.500	11.914	-16.449	16.399	10.606	16.449	0.202	0.111	0.217
	15.2	-10.717	13.323	-10.170	13.408	10.465	11.000	0.173	0.099	0.166
	19.1	6.837	17.893	10.860	16.130	9.954	12.868	0.395	0.362	0.288
	19.2	-3.921	16.822	-2.061	12.548	11.694	8.934	0.230	0.166	0.193
2019	5.1	11.506	11.254	9.365	13.297	8.991	9.365	0.274	0.209	0.191
	5.2	-1.286	12.081	0.368	8.799	8.260	6.812	0.162	0.157	0.130
	16.1	-12.353	16.794	-10.199	16.063	13.099	15.571	0.239	0.170	0.249
	16.2	6.748	10.341	8.437	10.286	6.677	8.965	0.276	0.258	0.189
	20.1	7.310	12.933	10.146	12.010	8.599	11.318	0.440	0.455	0.303
	20.2	1.204	17.595	5.673	12.215	12.559	7.950	0.244	0.187	0.208
	24.1	1.806	15.448	0.686	12.213	9.383	9.397	0.195	0.164	0.156
	24.2	-2.230	11.225	-0.020	7.858	8.210	4.748	0.130	0.115	0.092

Table B.9: Summary statistics of the metrics by segment obtained with Method 4 (integral-benchmark aggregation of regression-based transformation), using SPEC 20, for Case 3 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.322	13.921	2.672	9.425	10.210	5.979	0.170	0.124	0.162
	1.2	-0.373	14.639	1.215	12.885	6.599	12.288	0.281	0.166	0.249
	4.1	-6.284	16.656	-2.621	12.063	12.995	7.053	0.172	0.110	0.148
	4.2	-3.005	16.998	-0.154	13.598	10.438	12.325	0.284	0.230	0.300
	10.1	6.668	9.291	8.473	9.636	6.057	9.524	0.267	0.248	0.198
	10.2	9.976	8.332	11.178	10.989	6.888	11.398	0.342	0.292	0.303
	15.1	-17.533	13.416	-17.080	18.292	12.334	17.080	0.212	0.109	0.217
	15.2	-21.824	24.150	-14.161	24.166	21.732	14.653	0.227	0.135	0.177
	19.1	7.023	16.787	9.987	15.160	9.780	12.907	0.368	0.335	0.280
	19.2	-4.283	16.784	-3.022	12.912	11.361	9.748	0.239	0.163	0.207
2019	5.1	13.329	10.628	11.471	14.006	9.691	11.471	0.275	0.204	0.198
	5.2	-1.206	12.113	0.502	8.869	8.219	6.916	0.163	0.156	0.133
	16.1	-12.261	16.330	-10.292	15.899	12.624	15.791	0.237	0.165	0.237
	16.2	12.427	11.902	12.133	14.086	9.819	12.133	0.308	0.240	0.252
	20.1	6.439	14.531	9.379	12.628	9.484	12.448	0.448	0.469	0.299
	20.2	1.999	16.371	5.622	11.354	11.816	8.064	0.235	0.185	0.205
	24.1	4.564	13.572	7.124	11.732	7.976	9.678	0.192	0.160	0.152
	24.2	-2.191	11.196	0.350	7.902	8.115	4.806	0.129	0.111	0.092

Table B.10: Summary statistics of the metrics by segment obtained with Method 4 (integral-benchmark aggregation of regression-based transformation), using SPEC 20, for Case 1 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	19.706	14.816	25.718	21.391	11.779	25.718	0.138	0.079	0.163
	1.2	-2.178	34.780	-11.945	28.938	15.433	28.685	0.158	0.108	0.132
	4.1	1.344	20.401	5.154	16.617	9.806	23.265	0.083	0.054	0.094
	4.2	-7.627	45.641	-1.123	32.557	30.238	25.966	0.157	0.122	0.142
	10.1	30.433	19.341	27.572	30.433	19.341	27.572	0.198	0.145	0.169
	10.2	35.921	24.949	37.682	38.580	19.783	37.682	0.263	0.158	0.224
	15.1	-53.332	24.248	-56.802	53.332	24.248	56.802	0.177	0.074	0.196
	15.2	-72.574	78.825	-54.930	77.348	73.337	54.930	0.203	0.113	0.176
	19.1	23.808	40.695	34.565	40.689	19.641	43.895	0.227	0.139	0.164
	19.2	-2.914	22.450	8.681	17.217	12.965	12.390	0.088	0.052	0.080
2019	5.1	48.354	23.860	49.422	48.354	23.860	49.422	0.249	0.143	0.256
	5.2	3.777	30.295	17.134	22.536	18.491	19.496	0.103	0.063	0.116
	16.1	-26.221	41.513	-34.963	41.797	22.077	40.053	0.181	0.083	0.199
	16.2	37.851	46.460	37.780	45.662	37.383	37.780	0.213	0.140	0.197
	20.1	14.652	46.745	12.146	35.741	30.745	33.316	0.246	0.277	0.198
	20.2	27.018	22.498	17.481	27.018	22.498	17.481	0.176	0.171	0.091
	24.1	27.069	34.007	30.908	35.883	22.564	30.908	0.171	0.121	0.139
	24.2	8.697	20.097	18.599	19.019	8.374	18.599	0.094	0.048	0.096

Table B.11: Summary statistics of the metrics by segment obtained with Method 4 (integral-benchmark aggregation of regression-based transformation), using SPEC 20, for Case 2 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	17.543	14.116	22.699	19.427	10.863	22.699	0.125	0.075	0.140
	1.2	-3.552	37.665	-16.586	31.869	15.763	32.361	0.175	0.126	0.149
	4.1	-2.364	20.273	1.150	15.813	11.215	18.952	0.076	0.054	0.105
	4.2	-11.141	45.698	-5.317	33.400	30.523	22.577	0.157	0.115	0.143
	10.1	27.535	17.946	23.648	27.535	17.946	23.648	0.179	0.134	0.145
	10.2	33.780	25.340	35.209	36.667	20.121	35.209	0.250	0.158	0.210
	15.1	-53.688	25.851	-60.640	53.688	25.851	60.640	0.178	0.080	0.200
	15.2	-57.654	46.193	-55.814	62.442	37.996	55.814	0.199	0.089	0.197
	19.1	38.941	55.090	50.758	56.099	33.651	50.758	0.338	0.268	0.280
	19.2	2.523	17.466	12.875	15.455	5.817	14.136	0.082	0.024	0.088
2019	5.1	50.339	22.313	49.779	50.339	22.313	49.779	0.259	0.135	0.259
	5.2	2.957	30.463	16.500	22.275	18.956	18.997	0.101	0.064	0.113
	16.1	-27.809	40.902	-37.586	41.881	22.996	37.586	0.181	0.086	0.186
	16.2	3.860	37.558	-1.632	29.540	20.247	28.765	0.158	0.123	0.180
	20.1	14.925	47.338	22.979	36.645	30.562	30.943	0.252	0.273	0.184
	20.2	25.660	21.696	16.772	25.660	21.696	16.772	0.168	0.165	0.090
	24.1	27.444	34.270	30.648	35.512	24.122	30.648	0.170	0.128	0.138
	24.2	9.343	17.486	17.428	16.394	9.697	17.428	0.079	0.054	0.085

Table B.12: Summary statistics of the metrics by segment obtained with Method 4 (integral-benchmark aggregation of regression-based transformation), using SPEC 20, for Case 3 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	18.899	14.581	24.502	20.830	11.095	24.502	0.134	0.077	0.152
	1.2	-3.268	36.297	-14.527	30.622	15.359	30.444	0.167	0.113	0.140
	4.1	-1.017	20.642	2.830	16.490	10.491	20.721	0.081	0.053	0.104
	4.2	-9.647	46.265	-3.575	33.500	30.654	24.507	0.159	0.119	0.153
	10.1	28.749	18.771	25.608	28.749	18.771	25.608	0.187	0.140	0.157
	10.2	34.940	24.812	36.110	37.380	20.243	36.110	0.255	0.160	0.215
	15.1	-54.794	24.791	-59.700	54.794	24.791	59.700	0.182	0.076	0.203
	15.2	-71.201	76.695	-55.055	75.287	71.999	55.055	0.197	0.113	0.176
	19.1	26.182	41.623	36.005	43.254	18.696	45.228	0.244	0.140	0.218
	19.2	-2.244	20.931	7.729	15.856	12.275	13.034	0.081	0.051	0.083
2019	5.1	49.085	24.328	50.958	49.085	24.328	50.958	0.253	0.144	0.266
	5.2	3.060	30.603	16.809	22.390	19.042	19.355	0.101	0.064	0.115
	16.1	-28.419	40.409	-39.137	41.628	23.525	39.137	0.179	0.087	0.179
	16.2	32.472	37.533	36.343	41.570	24.956	36.343	0.200	0.111	0.215
	20.1	13.677	47.644	17.426	35.723	31.613	29.620	0.245	0.278	0.176
	20.2	25.255	21.020	17.002	25.255	21.020	17.002	0.166	0.161	0.093
	24.1	28.663	34.639	31.292	35.550	26.150	31.292	0.172	0.136	0.141
	24.2	8.845	16.762	16.585	15.865	8.866	16.585	0.077	0.049	0.079

Table B.13: Summary statistics of the metrics by segment obtained with Method 5 (smoothed interpolation approach using volume estimates directly obtained from individual bus passes), for Case 1 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-2.354	11.665	-3.235	9.073	7.557	6.506	0.203	0.194	0.135
	1.2	-0.834	15.234	0.413	12.110	9.046	10.107	0.261	0.203	0.210
	4.1	-1.118	9.640	-2.345	7.690	5.802	6.024	0.132	0.103	0.124
	4.2	-4.744	10.861	-1.834	8.803	7.855	6.126	0.157	0.112	0.158
	10.1	9.492	17.379	8.365	14.265	13.628	9.789	0.307	0.226	0.265
	10.2	3.886	12.064	5.639	10.236	7.265	8.589	0.298	0.269	0.255
	15.1	7.156	24.820	2.174	18.908	17.381	13.746	0.227	0.178	0.215
	15.2	27.981	34.753	15.533	31.365	31.635	24.988	0.323	0.286	0.299
	19.1	14.909	34.168	16.446	29.112	22.862	20.350	0.671	0.577	0.469
	19.2	-10.061	17.266	-10.708	15.988	11.804	13.573	0.296	0.193	0.255
2019	5.1	27.039	19.753	26.927	28.090	18.181	26.927	0.513	0.292	0.464
	5.2	1.695	14.881	0.115	11.211	9.771	9.765	0.200	0.154	0.183
	16.1	-11.206	24.272	-8.803	19.369	18.100	16.564	0.321	0.265	0.290
	16.2	31.861	51.871	10.208	35.773	49.175	11.084	0.558	0.635	0.229
	20.1	0.256	17.358	-0.077	12.405	11.972	9.649	0.310	0.280	0.298
	20.2	1.845	16.744	1.527	13.095	10.374	12.738	0.266	0.189	0.245
	24.1	18.324	20.504	18.536	22.304	15.922	20.521	0.361	0.285	0.293
	24.2	0.823	18.060	0.927	13.623	11.631	11.302	0.235	0.195	0.236

Table B.14: Summary statistics of the metrics by segment obtained with Method 5 (smoothed interpolation approach using volume estimates directly obtained from individual bus passes), for Case 2 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-2.354	11.665	-3.235	9.073	7.557	6.506	0.203	0.194	0.135
	1.2	0.058	15.039	1.130	11.621	9.330	9.044	0.241	0.199	0.194
	4.1	-1.118	9.640	-2.345	7.690	5.802	6.024	0.132	0.103	0.124
	4.2	-4.744	10.861	-1.834	8.803	7.855	6.126	0.157	0.112	0.158
	10.1	9.492	17.379	8.365	14.265	13.628	9.789	0.307	0.226	0.265
	10.2	5.126	10.592	6.321	9.482	6.803	8.410	0.274	0.260	0.249
	15.1	1.607	18.072	0.482	14.815	10.190	12.992	0.199	0.151	0.182
	15.2	4.553	15.431	4.118	10.928	11.607	6.753	0.179	0.223	0.101
	19.1	14.904	28.260	17.045	26.590	17.217	20.350	0.621	0.549	0.471
	19.2	-8.197	16.935	-6.090	14.548	11.751	12.720	0.265	0.188	0.245
2019	5.1	22.873	18.318	21.726	24.082	16.643	21.726	0.477	0.325	0.434
	5.2	1.695	14.881	0.115	11.211	9.771	9.765	0.200	0.154	0.183
	16.1	-11.206	24.272	-8.803	19.369	18.100	16.564	0.321	0.265	0.290
	16.2	5.530	15.143	4.518	10.548	12.077	8.791	0.221	0.200	0.163
	20.1	2.526	15.247	0.960	10.956	10.754	8.533	0.290	0.285	0.206
	20.2	1.053	16.148	0.018	12.365	10.234	11.613	0.259	0.194	0.231
	24.1	16.436	24.006	18.536	23.742	16.531	21.491	0.377	0.283	0.334
	24.2	1.456	17.469	0.927	13.306	11.162	11.302	0.227	0.167	0.236

Table B.15: Summary statistics of the metrics by segment obtained with Method 5 (smoothed interpolation approach using volume estimates directly obtained from individual bus passes), for Case 3 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-2.349	11.666	-3.231	9.071	7.559	6.502	0.203	0.194	0.135
	1.2	-0.814	15.223	0.419	12.097	9.044	10.100	0.261	0.202	0.209
	4.1	-1.112	9.641	-2.340	7.689	5.803	6.031	0.132	0.103	0.124
	4.2	-4.739	10.860	-1.830	8.801	7.853	6.129	0.157	0.112	0.158
	10.1	9.498	17.381	8.371	14.269	13.630	9.790	0.307	0.226	0.265
	10.2	3.906	12.033	5.643	10.223	7.243	8.594	0.297	0.269	0.255
	15.1	4.089	19.128	2.183	15.835	11.225	13.756	0.203	0.153	0.175
	15.2	11.560	19.377	7.377	15.728	16.071	9.000	0.207	0.242	0.079
	19.1	13.853	31.777	16.455	27.978	20.010	20.344	0.650	0.549	0.468
	19.2	-10.038	17.249	-10.695	15.967	11.787	13.570	0.295	0.193	0.254
2019	5.1	25.551	18.646	21.733	26.601	17.067	21.733	0.493	0.288	0.463
	5.2	1.702	14.883	0.122	11.213	9.773	9.766	0.200	0.154	0.183
	16.1	-11.200	24.274	-8.796	19.369	18.100	16.566	0.321	0.265	0.290
	16.2	20.566	30.089	10.215	24.470	26.914	10.973	0.408	0.359	0.230
	20.1	0.270	17.340	-0.074	12.397	11.955	9.649	0.310	0.280	0.298
	20.2	1.642	16.453	1.531	12.887	10.138	12.736	0.264	0.189	0.232
	24.1	18.197	20.606	18.544	22.175	16.088	20.531	0.359	0.287	0.293
	24.2	0.849	18.039	0.933	13.615	11.610	11.303	0.235	0.194	0.236

Table B.16: Summary statistics of the metrics by segment obtained with Method 5 (smoothed interpolation approach using volume estimates directly obtained from individual bus passes), for Case 1 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-12.545	31.875	-9.399	27.004	18.680	25.636	0.176	0.141	0.161
	1.2	-17.930	33.695	-23.976	32.280	17.169	31.642	0.173	0.091	0.151
	4.1	-5.195	18.731	-12.097	16.426	8.220	16.812	0.078	0.035	0.071
	4.2	-20.794	18.571	-16.267	20.794	18.571	16.267	0.095	0.062	0.115
	10.1	23.696	19.912	21.707	24.341	18.983	21.707	0.142	0.105	0.131
	10.2	13.629	45.908	26.845	39.200	23.055	38.024	0.256	0.146	0.226
	15.1	18.291	44.938	12.326	33.830	32.781	14.595	0.116	0.118	0.047
	15.2	68.010	77.500	73.032	86.779	51.155	73.032	0.253	0.082	0.253
	19.1	53.364	60.124	79.865	67.142	40.961	79.865	0.372	0.222	0.502
	19.2	-41.824	18.997	-46.515	41.824	18.997	46.515	0.229	0.108	0.240
2019	5.1	88.295	39.904	83.743	88.295	39.904	83.743	0.444	0.207	0.459
	5.2	8.653	41.845	-0.863	28.028	30.362	21.214	0.124	0.124	0.131
	16.1	-37.663	69.274	-48.399	69.868	27.543	68.329	0.309	0.126	0.340
	16.2	101.723	166.107	28.979	112.604	157.705	38.083	0.424	0.522	0.171
	20.1	-11.298	29.965	-16.724	26.935	14.158	31.543	0.171	0.113	0.150
	20.2	11.278	38.705	13.409	34.106	17.011	35.057	0.206	0.109	0.220
	24.1	66.920	55.000	71.651	76.636	37.387	71.651	0.349	0.198	0.323
	24.2	18.872	44.381	24.971	39.857	23.062	36.028	0.196	0.134	0.172

Table B.17: Summary statistics of the metrics by segment obtained with Method 5 (smoothed interpolation approach using volume estimates directly obtained from individual bus passes), for Case 2 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-12.545	31.875	-9.399	27.004	18.680	25.636	0.176	0.141	0.161
	1.2	-14.245	34.079	-23.976	29.835	18.964	31.642	0.154	0.099	0.151
	4.1	-5.195	18.731	-12.097	16.426	8.220	16.812	0.078	0.035	0.071
	4.2	-20.794	18.571	-16.267	20.794	18.571	16.267	0.095	0.062	0.115
	10.1	23.696	19.912	21.707	24.341	18.983	21.707	0.142	0.105	0.131
	10.2	13.629	45.908	26.845	39.200	23.055	38.024	0.256	0.146	0.226
	15.1	18.291	44.938	12.326	33.830	32.781	14.595	0.116	0.118	0.047
	15.2	-4.169	57.912	-2.478	43.433	33.331	55.226	0.155	0.130	0.166
	19.1	83.853	71.181	92.675	89.071	63.356	92.675	0.550	0.467	0.502
	19.2	-26.377	19.435	-26.282	26.377	19.435	26.282	0.148	0.114	0.155
2019	5.1	93.560	34.548	83.743	93.560	34.548	83.743	0.470	0.180	0.459
	5.2	8.653	41.845	-0.863	28.028	30.362	21.214	0.124	0.124	0.131
	16.1	-37.663	69.274	-48.399	69.868	27.543	68.329	0.309	0.126	0.340
	16.2	3.231	29.252	15.680	24.870	12.094	28.529	0.117	0.054	0.137
	20.1	-7.970	29.145	-16.724	25.102	13.728	20.536	0.157	0.107	0.150
	20.2	11.278	38.705	13.409	34.106	17.011	35.057	0.206	0.109	0.220
	24.1	66.920	55.000	71.651	76.636	37.387	71.651	0.349	0.198	0.323
	24.2	21.767	37.479	24.971	36.962	19.069	36.028	0.177	0.095	0.172

Table B.18: Summary statistics of the metrics by segment obtained with Method 5 (smoothed interpolation approach using volume estimates directly obtained from individual bus passes), for Case 3 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-12.545	31.875	-9.399	27.004	18.680	25.636	0.176	0.141	0.161
	1.2	-17.859	33.683	-23.976	32.208	17.216	31.642	0.173	0.091	0.151
	4.1	-5.195	18.731	-12.097	16.426	8.220	16.812	0.078	0.035	0.071
	4.2	-20.794	18.571	-16.267	20.794	18.571	16.267	0.095	0.062	0.115
	10.1	23.696	19.912	21.707	24.341	18.983	21.707	0.142	0.105	0.131
	10.2	13.629	45.908	26.845	39.200	23.055	38.024	0.256	0.146	0.226
	15.1	18.291	44.938	12.326	33.830	32.781	14.595	0.116	0.118	0.047
	15.2	39.706	50.544	55.858	58.475	20.120	60.927	0.196	0.098	0.195
	19.1	52.247	58.796	80.449	65.639	40.188	80.449	0.365	0.221	0.502
	19.2	-41.681	18.882	-46.515	41.681	18.882	46.515	0.229	0.107	0.239
2019	5.1	88.298	39.908	83.743	88.298	39.908	83.743	0.444	0.207	0.459
	5.2	8.653	41.845	-0.863	28.028	30.362	21.214	0.124	0.124	0.131
	16.1	-37.663	69.274	-48.399	69.868	27.543	68.329	0.309	0.126	0.340
	16.2	78.404	119.151	28.979	89.285	109.856	38.083	0.344	0.361	0.171
	20.1	-11.295	29.963	-16.724	26.932	14.156	31.543	0.171	0.113	0.150
	20.2	11.278	38.705	13.409	34.106	17.011	35.057	0.206	0.109	0.220
	24.1	66.920	55.000	71.651	76.636	37.387	71.651	0.349	0.198	0.323
	24.2	18.963	44.161	24.971	39.766	22.909	36.028	0.195	0.133	0.172

Table B.19: Summary statistics of the metrics by segment obtained with Method 6 (smoothed interpolation aggregation of regression-based transformations), using SPEC 20, for Case 1 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.460	14.527	3.264	9.795	10.703	6.814	0.175	0.124	0.172
	1.2	-0.388	14.933	1.947	12.928	7.147	11.886	0.280	0.168	0.271
	4.1	-6.656	17.162	-1.048	13.005	12.913	8.886	0.188	0.111	0.175
	4.2	-2.846	17.334	0.241	13.416	11.161	12.044	0.278	0.234	0.288
	10.1	7.244	9.636	9.437	10.176	6.361	9.714	0.282	0.254	0.205
	10.2	10.137	8.528	11.326	11.301	6.849	11.754	0.349	0.287	0.318
	15.1	-17.278	13.591	-17.110	18.144	12.380	17.110	0.210	0.107	0.227
	15.2	-22.156	24.826	-15.942	24.836	22.059	17.353	0.235	0.143	0.221
	19.1	6.255	17.180	8.645	15.029	10.128	12.023	0.361	0.342	0.217
	19.2	-4.160	16.276	-3.428	12.754	10.742	9.642	0.239	0.165	0.207
2019	5.1	12.741	8.959	12.267	12.953	8.641	12.267	0.259	0.183	0.267
	5.2	-0.727	10.972	1.083	8.075	7.351	6.085	0.146	0.123	0.109
	16.1	-11.793	17.343	-10.137	16.116	13.206	14.583	0.242	0.174	0.235
	16.2	15.623	19.627	10.577	17.523	17.899	10.577	0.344	0.282	0.263
	20.1	5.807	14.178	5.403	11.932	9.457	11.558	0.423	0.438	0.277
	20.2	1.581	16.813	5.205	11.274	12.435	7.868	0.231	0.202	0.175
	24.1	1.809	14.297	1.167	11.512	8.425	10.573	0.187	0.160	0.165
	24.2	-2.582	11.400	-1.042	8.011	8.402	5.043	0.132	0.113	0.092

Table B.20: Summary statistics of the metrics by segment obtained with Method 6 (smoothed interpolation aggregation of regression-based transformations), using SPEC 20, for Case 2 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.748	13.820	2.914	9.450	10.113	6.230	0.168	0.116	0.164
	1.2	-0.698	14.626	-0.825	12.975	6.400	12.310	0.281	0.162	0.246
	4.1	-7.203	16.178	-2.314	12.661	12.278	8.539	0.182	0.104	0.180
	4.2	-3.655	17.210	-0.821	13.421	11.204	11.693	0.274	0.225	0.274
	10.1	6.851	9.369	8.610	9.535	6.531	8.855	0.265	0.246	0.197
	10.2	9.919	8.228	10.972	11.066	6.541	12.218	0.342	0.279	0.301
	15.1	-15.757	11.889	-17.375	16.742	10.415	17.375	0.206	0.109	0.225
	15.2	-10.877	14.388	-9.745	14.368	10.709	10.517	0.189	0.109	0.171
	19.1	6.525	18.424	7.713	16.381	10.278	15.297	0.398	0.367	0.264
	19.2	-4.254	16.206	-3.374	12.450	11.032	8.217	0.230	0.163	0.177
2019	5.1	11.634	11.010	11.858	13.227	8.965	11.858	0.272	0.204	0.233
	5.2	-0.988	11.067	0.597	8.137	7.454	5.889	0.146	0.122	0.102
	16.1	-11.951	16.724	-10.417	15.963	12.743	15.167	0.240	0.169	0.226
	16.2	5.372	10.275	6.181	9.301	6.777	7.589	0.252	0.239	0.159
	20.1	7.284	12.582	9.359	11.958	8.116	11.176	0.440	0.453	0.268
	20.2	1.150	16.953	5.836	11.289	12.561	6.957	0.228	0.196	0.181
	24.1	1.633	16.081	1.266	12.614	9.854	10.328	0.202	0.169	0.180
	24.2	-2.329	11.487	-0.145	8.006	8.448	5.314	0.133	0.116	0.097

Table B.21: Summary statistics of the metrics by segment obtained with Method 6 (smoothed interpolation aggregation of regression-based transformations), using SPEC 20, for Case 3 and Data set, for 15-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	-1.632	14.304	2.993	9.700	10.515	6.658	0.173	0.121	0.171
	1.2	-0.486	14.758	1.189	12.927	6.784	12.031	0.280	0.162	0.261
	4.1	-7.110	16.792	-1.845	12.961	12.717	9.049	0.186	0.107	0.179
	4.2	-3.324	17.429	-0.331	13.573	11.250	11.885	0.279	0.231	0.289
	10.1	6.948	9.516	8.941	9.859	6.353	9.297	0.273	0.250	0.197
	10.2	9.938	8.303	11.228	11.008	6.764	11.991	0.340	0.283	0.304
	15.1	-17.515	13.351	-17.582	18.328	12.180	17.582	0.213	0.108	0.231
	15.2	-21.838	24.427	-15.437	24.536	21.628	17.250	0.233	0.139	0.221
	19.1	6.684	17.178	9.190	15.323	9.951	12.990	0.370	0.341	0.250
	19.2	-4.355	16.166	-2.719	12.527	10.929	9.146	0.233	0.165	0.189
2019	5.1	13.434	10.255	12.328	13.669	9.931	12.328	0.268	0.195	0.259
	5.2	-1.006	11.170	0.409	8.223	7.514	5.790	0.147	0.122	0.101
	16.1	-11.879	16.202	-10.467	15.752	12.263	15.481	0.238	0.163	0.213
	16.2	13.445	14.935	10.955	15.486	12.739	10.955	0.321	0.244	0.274
	20.1	6.346	14.297	8.541	12.506	9.227	12.651	0.445	0.468	0.273
	20.2	1.736	16.120	6.023	10.610	12.136	6.752	0.221	0.195	0.170
	24.1	3.504	14.265	2.233	11.916	8.342	9.917	0.196	0.165	0.166
	24.2	-2.298	11.469	0.296	8.050	8.372	5.369	0.131	0.113	0.099

Table B.22: Summary statistics of the metrics by segment obtained with Method 6 (smoothed interpolation aggregation of regression-based transformations), using SPEC 20, for Case 1 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	19.686	14.020	25.243	20.699	12.200	25.243	0.133	0.079	0.166
	1.2	-2.095	35.363	-11.869	29.091	16.380	28.976	0.160	0.118	0.134
	4.1	-0.249	23.301	2.372	18.731	11.562	25.902	0.092	0.061	0.114
	4.2	-7.522	45.522	0.530	31.689	31.091	23.632	0.151	0.120	0.138
	10.1	32.150	17.370	30.684	32.150	17.370	30.684	0.206	0.135	0.188
	10.2	35.852	24.472	38.542	38.043	20.248	38.542	0.260	0.164	0.229
	15.1	-49.348	23.140	-55.408	49.348	23.140	55.408	0.163	0.071	0.182
	15.2	-72.611	79.561	-49.013	78.293	73.004	49.013	0.206	0.111	0.157
	19.1	23.948	38.672	37.547	39.359	18.901	44.993	0.219	0.127	0.177
	19.2	-3.207	17.718	1.240	13.313	10.915	10.528	0.068	0.046	0.067
2019	5.1	49.194	23.393	56.557	49.194	23.393	56.557	0.253	0.141	0.257
	5.2	2.289	33.845	18.326	24.874	20.731	21.726	0.111	0.068	0.123
	16.1	-27.062	41.001	-34.428	42.655	20.316	40.018	0.185	0.077	0.199
	16.2	47.901	60.613	36.105	56.315	51.486	36.105	0.249	0.161	0.214
	20.1	13.999	45.916	20.050	35.125	29.959	32.099	0.242	0.273	0.191
	20.2	25.161	23.595	13.322	25.161	23.595	13.322	0.166	0.177	0.059
	24.1	26.245	35.623	30.820	36.701	22.386	30.820	0.175	0.120	0.140
	24.2	8.784	20.041	18.380	18.924	8.594	18.380	0.094	0.049	0.095

Table B.23: Summary statistics of the metrics by segment obtained with Method 6 (smoothed interpolation aggregation of regression-based transformations), using SPEC 20, for Case 2 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	17.519	13.329	22.781	18.814	11.081	22.781	0.121	0.074	0.144
	1.2	-3.473	37.978	-16.514	32.032	16.105	32.645	0.177	0.129	0.150
	4.1	-4.585	23.418	-1.529	18.734	12.786	21.177	0.089	0.059	0.118
	4.2	-11.395	45.664	-4.279	32.630	31.537	20.228	0.152	0.113	0.128
	10.1	28.836	16.904	26.530	28.836	16.904	26.530	0.186	0.128	0.163
	10.2	33.713	24.785	35.985	36.192	20.299	35.985	0.248	0.162	0.214
	15.1	-50.229	24.704	-56.445	50.229	24.704	56.445	0.166	0.077	0.186
	15.2	-57.964	49.538	-57.081	63.893	39.837	57.081	0.205	0.093	0.196
	19.1	38.975	55.176	49.646	56.179	33.682	49.646	0.338	0.270	0.274
	19.2	0.674	15.679	9.467	12.984	7.049	11.894	0.069	0.033	0.077
2019	5.1	51.111	21.719	56.506	51.111	21.719	56.506	0.262	0.132	0.260
	5.2	0.912	33.892	17.413	24.754	20.849	19.650	0.109	0.066	0.114
	16.1	-28.790	40.407	-37.089	42.879	21.323	37.408	0.185	0.080	0.186
	16.2	0.060	37.838	1.666	31.823	15.818	32.910	0.163	0.094	0.193
	20.1	14.410	47.035	27.055	36.263	30.337	29.749	0.250	0.272	0.197
	20.2	23.816	22.745	12.337	23.816	22.745	12.337	0.158	0.171	0.054
	24.1	26.638	35.910	30.586	36.290	24.104	30.586	0.174	0.127	0.138
	24.2	9.408	17.441	17.186	16.338	9.801	17.186	0.079	0.054	0.084

Table B.24: Summary statistics of the metrics by segment obtained with Method 6 (smoothed interpolation aggregation of regression-based transformations), using SPEC 20, for Case 3 and Data set, for 60-minute periods

Data Set	Segment	Difference			Absolute Difference			ARE		
		Mean`	Standard Deviation	Median	Mean	Standard Deviation	Median	Mean	Standard Deviation	Median
2018	1.1	18.877	13.766	24.428	20.203	11.361	24.428	0.130	0.076	0.155
	1.2	-3.191	36.745	-14.455	30.775	16.033	30.723	0.168	0.119	0.142
	4.1	-2.920	23.616	0.185	18.753	12.545	23.050	0.090	0.061	0.126
	4.2	-9.748	46.189	-2.316	32.651	31.631	22.230	0.154	0.118	0.141
	10.1	30.182	17.336	28.502	30.182	17.336	28.502	0.194	0.132	0.175
	10.2	34.874	24.335	36.898	36.893	20.569	36.898	0.253	0.165	0.220
	15.1	-51.220	23.739	-57.464	51.220	23.739	57.464	0.170	0.073	0.189
	15.2	-71.307	77.279	-47.792	76.314	71.482	47.792	0.201	0.110	0.153
	19.1	26.317	39.692	38.020	41.757	18.689	46.396	0.236	0.133	0.233
	19.2	-3.091	17.577	0.604	12.724	11.454	11.692	0.065	0.050	0.064
2019	5.1	48.567	25.283	57.325	48.567	25.283	57.325	0.250	0.148	0.266
	5.2	0.509	33.940	17.380	24.845	20.787	18.584	0.109	0.065	0.111
	16.1	-29.528	39.936	-38.674	42.754	21.901	38.674	0.184	0.081	0.179
	16.2	48.185	59.156	34.499	57.167	48.944	34.499	0.252	0.152	0.227
	20.1	12.584	46.905	22.141	34.876	31.078	28.447	0.240	0.276	0.169
	20.2	23.421	22.027	12.312	23.421	22.027	12.312	0.156	0.166	0.059
	24.1	27.872	36.302	31.253	36.422	26.011	31.253	0.175	0.135	0.141
	24.2	8.913	16.703	16.368	15.769	9.031	16.368	0.077	0.050	0.078