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Vehicle-based Panhandle Bridge Monitoring

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Title: **An expectation-maximization-based framework for vehicle-vibration-based indirect structural health monitoring of bridges**

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ABSTRACT

We propose a vehicle-vibration-based indirect structural health monitoring (SHM) framework that uses acceleration signals collected from within a moving vehicle to identify global modal and structural parameters of a full-scale and in-service bridge. Motivated by many benefits of indirect sensing methods, such as low-cost, low-maintenance and no interruption to traffic, researchers have in the past presented different algorithms and evaluated them on several simulation and lab-scale datasets. However, the uncertainties of the real-world vehicle-bridge interaction system and limited training data may cause previous methods to fail on full-scale bridges. To address these uncertainties, we 1) cast the vehicle-bridge interaction system as a linear time-varying Gaussian state-space model, which is not only able to estimate unobserved bridge responses but also able to add a stochastic process for modeling uncertainties, and 2) propose a hybrid algorithm that uses non-linear least squares and the expectation-maximization algorithm to estimate modal and structural parameters of the bridge using partially observed data (only the vehicle's dynamic response is observed). We conducted field experiments on a steel truss bridge carrying two rail lines across the Monongahela River in Pittsburgh, Pennsylvania. For estimating the damage that is simulated by placing stationary trains on the bridge, our proposed approach has a 36.3% error reduction compared to a fully data-driven method. The results show that our proposed algorithm provides a potentially practical approach for continuous monitoring of in-service bridges.

INTRODUCTION

The U.S. has 614,387 bridges, 9.1% of which were structurally deficient in 2016 [1]. As many bridges approach the end of their designed service life, the number of deficient bridges continues to increase quickly. Hence, accurate and efficient bridge health monitoring approaches are needed to improve the nation-wide bridge management.

Conventional bridge SHM techniques utilize sensing data directly collected from the structure of interest to estimate the physical parameters (e.g.mode shapes, mass, etc). These direct approaches pose some challenges including expensive sensing systems, labor-intensive sensor deployment and maintenance, and unstable power supply [2].

Alternatively, indirect SHM of bridges collects vehicle-vibration signals when the

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vehicles pass through the target infrastructure [3–5]. The bridge's physical parameters are estimated by analyzing the changes of vehicle vibrations induced by the bridge. This indirect SHM sensing system does not require intensive deployment and maintenance. It has no interruption to the regular transportation service for initial instrumentation and maintenance, and it can be powered by the vehicular electrical system. Despite its drawbacks, such as higher uncertainties, indirect SHM has gained increasing popularity.

Previous work on indirect SHM concentrates on finding predictive features from the dynamic response of the moving vehicle to diagnose bridge damage. These methods mainly fall into two categories: modal analysis, and pure data-driven approaches. Modal analysis focuses on extracting modal parameters including natural frequencies [3, 6] and mode shapes [7] of the bridge. Fully data-driven approaches use signal processing and machine learning techniques to extract predictive features for diagnosing damage [8, 9].

However, both of these approaches have unsolved problems for achieving indirect SHM of full-scale bridges. For the modal analysis methods, system and environmental uncertainties make the identification and reconstruction of modal parameters difficult and inaccurate [10]. To ensure adequate performance, data-driven indirect SHM approaches require a large set of training data, which is expensive to obtain in practice [9]. In addition, the limited physical interpretability of these approaches constrains the application of the learned model on other bridges. We encountered these challenges in our field experiment on a 100 years old truss bridge.

To address these challenges, we represent the dynamic vehicle-bridge interaction (VBI) system as a state-space model. Conventionally, a dynamical system can be represented in four different ways: the state-space model, the differential equation model, the impulse response model, and the transfer function model [11]. We choose the state-space model for two reasons. First, the state-space representation of the VBI system models system and environment noises as stochastic processes. This enables a robust estimation of system parameters regardless of various uncertainties. Second, the state-space representation embeds the physical understanding of the VBI system to model the observed vehicle response and the unobserved bridge response as state variables. When there is limited training data, using physical knowledge as regularization help improve the estimation of bridge response and system parameters.

With the state-space representation, the objective of the indirect SHM becomes one of parameter estimation of a stochastic dynamical system, which has been studied for several decades [11]. Least squares methods [12] and artificial neural networks [13], which minimize the error between observed system responses and estimated responses have provided reasonable system identification results. Stochastic subspace identification methods, which estimate jointly the state variables and system parameters using observed state have been applied to extract bridge frequencies from the collected dynamic response of a moving vehicle [6]. However, when the model depends on unobserved variables, the expectation-maximization (EM) algorithm which iteratively computes maximum likelihood is more useful for system identification. This is because the inference and learning processes of EM are optimal in a probabilistic sense [14].

In this paper, we first transform the differential equation model of an idealized VBI system to a linear time-varying Gaussian state-space model with incomplete observations and structured transition matrices. Then, we propose a hybrid algorithm that uses non-linear least squares to pre-estimate modal and structural parameters of the VBI sys-

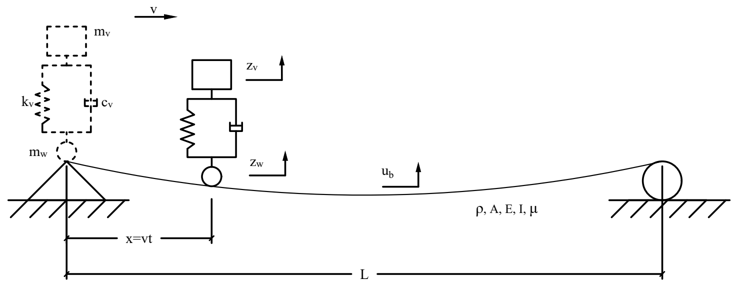

Figure 1. Model of a two-degree of freedom oscillator moving over a single-span, simply supported beam with a constant velocity.

tem and updates the parameters and uncertainty co-variance matrices by using the EM algorithm. In the maximization stage of EM, maximization of the likelihood function has no closed-form solutions due to the time-varying property and structured transition matrices of the VBI system. Also, due to the non-convex likelihood function, most gradient-based optimization methods can get stuck on local optima. Hence, to eliminate unrealistic local optima and accelerate convergence, we constrain the space of the parameters by using the domain knowledge of the bridge. We then apply the proposed methodology to an actual bridge-vehicle system. Though the EM algorithm with domain knowledge for system identification has been studied before, to the best of our knowledge this is the first time that it is applied in the context of a VBI system and more importantly, the first time that it is evaluated on a full-scale bridge under operation.

The rest of the paper consists of three sections: derivation of the state-space representation of the idealized VBI system; expectation-maximization algorithm with structural domain knowledge for estimating bridge parameters of the VBI system; validation and evaluation of the proposed algorithm on simulation data and a real-world dataset.

THE VEHICLE-BRIDGE INTERACTION SYSTEM

In this section, we first derive the differential equation model of the VBI system, idealized as a sprung mass system traveling with a constant speed on a simple supported beam. Then, we introduce a Gaussian state-space model of the VBI system based on the differential equation model and characterize the properties of this state-space model.

Differential equation model of the VBI system

We derive the theoretical formulation of the VBI system with the following assumptions: the beam is of the Euler-Bernoulli type with a constant cross-section; the vehicle wheel mass is zero; the wheel is always attached to the beam, which means that the displacement of the wheel is equal to that of the beam at the contact location; for convenience, the motion in physical coordinates is interpreted as a combination of the motions in each natural mode; and for the homogeneous simply supported beam, the mode shapes of the beam are sinusoidal functions.

The equations of motion for the VBI system (as shown in Figure 1) are:

$$
\begin{aligned}
&m_v \ddot{z}_v(t) + c_v \dot{z}_v(t) + k_v z_v(t) = k_v z_w(t) + c_v \dot{z}_w(t), \\
&EI \frac{\partial^4 u_b(x,t)}{\partial x^4} + \rho A \frac{\partial^2 u_b(x,t)}{\partial t^2} + \mu \frac{\partial u_b(x,t)}{\partial t} = -F_c(t)\delta(x - vt),
\end{aligned}
\tag{1}
$$

where t is time; x is the longitudinal coordinate of the beam with the origin at the left support; $m_v,\ k_v,\ c_v,\ z_v(t),\ z_w(t),\ v$ are the mass, stiffness, damping coefficient, vertical dynamic displacement of the vehicle chassis and the wheel, and vehicle velocity, respectively; $\rho,\ A,\ E,\ I,\ \mu,\ u_b(x,t)$ are the density, sectional area, Young's modulus, moment of inertia, damping coefficient, and vertical dynamic displacement of the beam, respectively; $\delta(x-vt)$ is the Dirac delta function; and $F_c(t)$ is the contact force:

$$F_c(t) = (m_v + m_w)g + m_v\ddot{z}_v(t) + m_w\ddot{z}_w(t), \tag{2}$$

where g is the gravity acceleration.

Using modal superposition, we have

$$u_b(x,t) = \sum_{n=1}^{\infty} \phi_n(x)q_{bn}(t) = \sum_{n=1}^{\infty} \big[\sqrt{\tfrac{2}{\rho AL}} \sin \tfrac{n\pi x}{L} q_{bn}(t)\big], \tag{3}$$

where n is the number of mode; $\phi_n(x) = \sqrt{\frac{2}{\rho AL}} \sin \frac{n\pi x}{L}$, $q_{bn}(t)$ are the n-th mode shape and dimensionless modal coordinate of the beam, respectively. Using Equations 2 and 3, and the assumptions that $m_w = 0,\ z_w(t) = u_b(vt,t)$, Equation 1 can be re-written as:

$$\begin{aligned} m_v\ddot{z}_v(t) + c_v\dot{z}_v(t) + k_v z_v(t) =& k_v \sum_{n=1}^{\infty} \phi_n(vt)q_{bn}(t) + c_v\Big(\sum_{n=1}^{\infty} \dot{\phi}_n(vt)q_{bn}(t) + \sum_{n=1}^{\infty} \phi_n(vt)\dot{q}_{bn}(t)\Big) \\ \ddot{q}_{bn}(t) + 2\xi_n\omega_n\dot{q}_{bn}(t) + \omega_n^2 q_{bn}(t) =& -\phi_n(vt)(m_v g + m_v\ddot{z}_v) \end{aligned} \tag{4}$$

where $\omega_n = \sqrt{\frac{EI}{\rho A}(\frac{n\pi}{L})^4}$, $\xi_n = \frac{\mu}{2\omega_n\rho A}$ are the n-th mode natural frequency and damping ratio of the beam, respectively.

State-space model of the VBI system

By rearranging Equation 4 and discretizing the continuous-time model by Euler's method [15], we obtain the following state-space representation of the VBI system:

$$\begin{aligned} \mathbf{z}_{k+1} &= \mathbf{A}_k\mathbf{z}_k + \mathbf{u}_k + \epsilon_k \\ \mathbf{y}_k &= \mathbf{C}\mathbf{z}_k + \eta_k \end{aligned} \tag{5}$$

where $k \in \{0, 1, 2, \cdots, T\}$ indicates the k-th sample (T is the total number of samples); $\mathbf{z}_k \in \mathbb{R}^{(2n+2)\times 1}$, $\mathbf{y}_k \in \mathbb{R}^{2\times 1}$ and $\mathbf{u}_k \in \mathbb{R}^{(2n+2)\times 1}$ are the state, observation and input at k, respectively; $\mathbf{A}_k \in \mathbb{R}^{(2n+2)\times(2n+2)}$ and $\mathbf{C} \in \mathbb{R}^{2\times(2n+2)}$ are the transition matrix and observation matrix at k, respectively; $\epsilon_k \sim \mathcal{N}(0, \mathbf{Q})$ and $\eta_k \sim \mathcal{N}(0, \mathbf{R})$ are Gaussian process noise and Gaussian observation noise of the system. $\mathbf{Q} \in \mathbb{R}^{(2n+2)\times(2n+2)}$ and $\mathbf{R} \in \mathbb{R}^{2\times 2}$ are covariance matrices.

In this work, we assume that the bridge vibrates in its fundamental mode allowing the state-space model to be expressed as:

$$\begin{aligned} \begin{bmatrix} z_{v,k+1} \\ \dot{z}_{v,k+1} \\ q_{b1,k+1} \\ \dot{q}_{b1,k+1} \end{bmatrix} =& \begin{bmatrix} 1, & \Delta t, & 0, & 0 \\ -\frac{k_v}{m_v}\Delta t, & 1-\frac{c_v}{m_v}\Delta t, & \begin{array}{c}\Delta t\big(\frac{k_v}{m_v}\phi_1(vk) \\ +\frac{c_v}{m_v}\dot{\phi}_1(vk)\big),\end{array} & \frac{c_v}{m_v}\Delta t\phi_1(vk) \\ 0, & 0, & 1, & \Delta t \\ k_v\Delta t\phi_1(vk), & c_v\Delta t\phi_1(vk), & \begin{array}{c}-\omega_1\Delta t + \Delta t\big[k_v\phi_1(vk)^2 \\ +c_v\phi_1(vk)\dot{\phi}_1(vk)\big],\end{array} & \begin{array}{c}1-2\xi_1\omega_1\Delta t \\ +c_v\Delta t\phi_1(vk)^2\end{array} \end{bmatrix} \begin{bmatrix} z_{v,k} \\ \dot{z}_{v,k} \\ q_{b1,k} \\ \dot{q}_{b1,k} \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ m_v g\Delta t\phi_1(vk) \end{bmatrix} + \epsilon_k, \\ \begin{bmatrix} y_{1,k} \\ y_{2,k} \end{bmatrix} =& \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_{v,k} \\ \dot{z}_{v,k} \\ q_{b1,k} \\ \dot{q}_{b1,k} \end{bmatrix} + \eta_k \end{aligned} \tag{6}$$

Equation 6 shows that the VBI system is a linear time-varying system where the transition matrices depend on time and are defined by the vehicle and bridge properties. Also, the observation matrix of the state-space representation of the VBI system is a non-square matrix where the column rank is not full. Thus, the observation of the system only contains the dynamic response of the moving vehicle.

EXPECTATION-MAXIMIZATION ALGORITHM WITH DOMAIN KNOWLEDGE FOR ESTIMATING PARAMETERS OF THE VBI SYSTEM

As mentioned in the previous section, the state-space representation of the VBI system depends on unobserved latent variables, which are the dynamic responses of the beam. In this incomplete-data situation, the expectation-maximization (EM) algorithm is commonly used to maximize the likelihood function to estimate parameters of the state-space model. Each iteration of the EM algorithm consists of an expectation-step (E-step) and a Maximization-step (M-step). In the E-step, the unobserved data are estimated given a current estimation of the model parameters and the observation. In the M-step, the model parameters are updated by maximizing the likelihood function given the observed data and estimated unobserved data [14].

For our problem, we want to estimate the bridge parameters (ω_1, ρA and ξ_1) given observations ($\mathbf{Y} = \mathbf{y}_{1:T}$) and the vehicle's parameters (m_v, k_v, and c_v). The expected likelihood function of the state-space model of the VBI system depends on $\mathbb{E}_{\mathbf{z}|\mathbf{y}}[\mathbf{z}_k\mathbf{z}_k^T|\mathbf{Y}]$, $\mathbb{E}_{\mathbf{z}|\mathbf{y}}[\mathbf{z}_k|\mathbf{Y}]$ and $\mathbb{E}_{\mathbf{z}|\mathbf{y}}[\mathbf{z}_k\mathbf{z}_{k+1}^T|\mathbf{Y}]$, which can be calculated by Kalman smoother [16] in the E-step. Then, because the system is time-varying and has structured transition matrices, updating the bridge parameters by maximizing the likelihood function does not have a closed-form expression and local optima exist. To address these challenges, we use least squares [12] to pre-estimate the parameters and constrain the parameters in ranges informed by structural domain knowledge (i.e., $\omega_1 \in [\omega_l, \omega_u],\ \rho A \in [\rho A_l, \rho A_u],\ \xi_1 \in [\xi_l, \xi_u]$). For example, the damping ratio is greater than zero and smaller than one ($\xi_1 \in [0, 1]$) as the bridge is under-damped. The proposed algorithm is presented in Table 1.

EXPERIMENTS AND RESULTS

In this section, we first validate the proposed algorithm on simulated data. We then present details of a field experiment on a full-scale truss bridge and evaluate the proposed algorithm on the field experimental data.

Simulation validation

To validate the ability of the proposed algorithm for identifying parameters of a VBI system, we first used a finite element model to create the vertical displacement and velocity of an oscillator that moves across a simply supported beam. The physical properties of the simulated VBI system are $m_v = 5.0 \times 10^4$ kg, $c_v = 0$ Ns/m, $k_v = 2.0 \times 10^6$ N/m, $v = 8.0$ m/s, $L = 80.0$ m, $\omega_1 = 18.2$ rad/s, $\rho A = 2.1 \times 10^4$ kg/m, $\xi_1 = 1.0 \times 10^{-3}$. Figure 2 shows the simulated vertical displacement and velocity and their fitted results by our algorithm. The estimated bridge properties using our algorithm are $\hat{\omega}_1 = 17.8$ rad/s, $\hat{\rho A} = 2.2 \times 10^4$ kg/m, $\hat{\xi}_1 = 1.0 \times 10^{-3}$, which are close to the true parameters.

Experimental setup and dataset

Our group conducted a field experiment on the Panhandle bridge that carries two rail

Algorithm 1 EM-algorithm with structural domain knowledge for indirect SHM

Require: Initialize known parameters: $m_v, k_v, c_v, v, L, \Delta t, \mathbf{C}$; structural domain constraints: $\omega_l, \omega_u, \rho A_l, \rho A_u, \xi_l, \xi_u$; unknown parameters: $\mathbf{z}_{0|0}$, $\mathbf{\Sigma}_{0|0}$, and $\mathbf{R}$, $\mathbf{Q}$.

Input: vertical displacement and velocity of the moving vehicle: $\mathbf{y}^*$

Pre-estimate $\hat{\omega}_1, \hat{\rho A}, \hat{\xi}_1$ by non-linear least squares:

$$\hat{\omega}_1, \hat{\rho A}, \hat{\xi}_1 = \arg \min_{\hat{\omega}_1, \hat{\rho A}, \hat{\xi}_1} \sum_{k=0}^{T} \|\mathbf{y}_k^* - \mathbf{C}\big[\mathbf{A}_k \mathbf{z}_k + \mathbf{u}_k\big]\|_2^2$$
$$\text{s.t. } \omega_1 \in [\omega_l, \omega_u],\ \rho A \in [\rho A_l, \rho A_u],\ \xi_1 \in [\xi_l, \xi_u]$$

Construct $\hat{\mathbf{A}}_1, \cdots, \hat{\mathbf{A}}_T$ and $\hat{\mathbf{u}}_1, \cdots, \hat{\mathbf{u}}_T$ using $\hat{\omega}_1, \hat{\rho A}, \hat{\xi}_1$.

while expected log-likelihood does not converge **do**

E-step:calculate $\mathbb{E}_{\mathbf{z}|\mathbf{y}}[\mathbf{z}_k \mathbf{z}_k^T|\mathbf{Y}], \mathbb{E}_{\mathbf{z}|\mathbf{y}}[\mathbf{z}_k|\mathbf{Y}], \mathbb{E}_{\mathbf{z}|\mathbf{y}}[\mathbf{z}_k \mathbf{z}_{k+1}^T|\mathbf{Y}]$ by Kalman smoother

M-step:

$$\hat{\omega}_1, \hat{\rho A}, \hat{\xi}_1 = \arg \max_{\hat{\omega}_1, \hat{\rho A}, \hat{\xi}_1} \mathbb{E}_{\mathbf{z}|\mathbf{y}}\big[\log \mathbb{P}(\mathbf{Z}, \mathbf{Y}|\mathbf{A}_1, \cdots, \mathbf{A}_T, \mathbf{u}_1, \cdots \mathbf{u}_T, \mathbf{Q}, \mathbf{R})\big]$$
$$\text{s.t. } \omega_1 \in [\omega_l, \omega_u],\ \rho A \in [\rho A_l, \rho A_u],\ \xi_1 \in [\xi_l, \xi_u]$$

end while

Return $\hat{\omega}_1, \hat{\rho A}, \hat{\xi}_1$

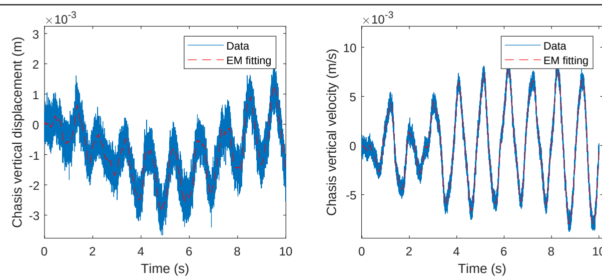

Figure 2. Simulated vertical displacement and velocity of the moving vehicle (blue curves) and the fitting results of them by the EM-algorithm (red curves).

lines of the Port Authority light rail system across the Monongahela River in Pittsburgh. We used one or two trains as a proxy for change in the weight (or density) of the bridge and positioned them along one of the tracks on the bridge. The other track was used for running another train back and forth. Sensors were mounted on the running train.

The Panhandle Bridge is a steel truss bridge built more than one hundred years ago. Our objective is to monitor the main span (112 meters) using the indirect SHM. Before the main experiments, we did free vibration tests on the main span and the moving light rail vehicle for obtaining their natural frequencies and damping ratios. The first dominant frequencies of the main bridge span and the moving vehicle are 1.515 Hz and 1.945 Hz, respectively, and the estimated damping ratios of the span and the vehicle are 0.008 and 0.188, respectively. We also know that the weight of the light rail vehicle is 5×10^4 kg.

The experiment consist of three trials with different loading methods:

- Trial one: complete 10 runs at 20 mph with an unloaded track. For each run, the instrumented light rail vehicle passed over the bridge at constant speed;
- Trial two: complete 10 runs at 20 mph, with a single train car located at 1/2 span from the beginning of the main track span.
- Trial three: complete 10 runs at 20 mph, with two train cars located at 1/3 and 2/3 span from the beginning of the main track span.

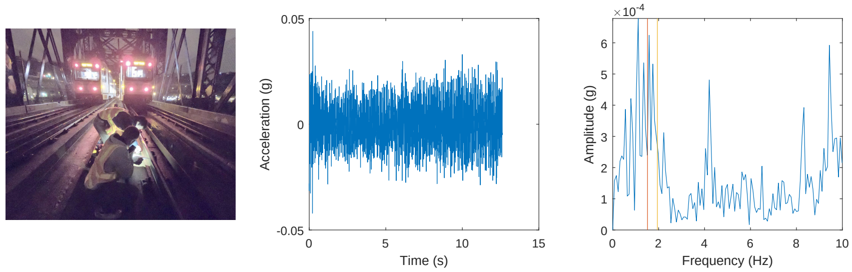

Figure 3. Picture of the experiment and an acceleration record collected from the vehicle while was passing over the bridge. The second subplot shows the raw acceleration signal, and the third subplot presents the amplitude spectrum of the acceleration. The vertical line in red and yellow indicate the natural frequency of the vehicle and the bridge, respectively.

During the experiment, we collected 30 vertical acceleration records (10 records for each trial) at the sampling rate of 2048 Hz from the light rail vehicle while it was moving across the bridge. Figure 3 shows one example of the raw acceleration signal and its amplitude spectrum. We do not observe large amplitude at the natural frequency of the bridge, which suggests that traditional modal identification methods would fail.

Experimental results

We applied the proposed algorithm to the field experimental data. We converted acceleration signals collected from the moving vehicle to velocity and displacement as input to the proposed algorithm. Figure 4 shows the results of the proposed algorithm on the bridge under no-loading (0 kg), one-car loading (5.0×10^4 kg) and two-car loading (1.0×10^5 kg) conditions. For the no-loading experiments, the root mean squared error (RMSE) for estimating the dominant frequencies of the bridge between the proposed algorithm and the free vibration test (1.945 Hz) is 0.278 Hz. We note that because of experimental noise, the estimation of bridge's natural frequency with one-car loading is not accurate and has large variance. Furthermore, by comparing the estimations for no-loading and two-car loading experiments, we can observe that heavier loading cases have larger density estimations and lower dominant frequency estimations, which agrees with the approximate definition of the natural frequency of the bridge ($\omega_1 = \sqrt{\frac{EI}{\rho A}\left(\frac{\pi}{L}\right)^4}$). Because adding dead load simulates change in the unit length weight (or uniform density) of the bridge, we normalize the estimated unit length density and use it to estimate the dead load on the bridge. The RMSE of the estimation using our algorithm is 3.5×10^4 kg. The RMSE of the estimation of the dead load by using a data-driven method [9] on the same dataset is 5.5×10^4 kg. Compared to the data-driven indirect SHM method, our algorithm reduces the estimation error by 36.3%.

CONCLUDING REMARKS

In this paper, we proposed an indirect SHM approach after casting the VBI system as a linear time-varying Gaussian state-space model. To address the challenges of parameter estimation of the VBI system with incomplete data, we used a hybrid algorithm that uses non-linear least squares to pre-estimate the parameters and applies the expectation-maximization algorithm to handle the incomplete-data situation. We evaluated the proposed algorithm on a field dataset collected from a light rail vehicle that passed over a full-scale steel truss bridge. Compared to a data-driven method, our proposed approach improves the error of the bridge loading estimation by 36.3%.

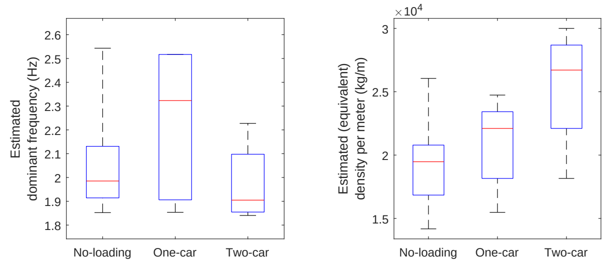

Figure 4. Evaluation results of our proposed approach. The first and the second subplots show boxplots of the estimated dominant frequency and unit length density of the bridge. The red dashed line indicates the natural frequency of the bridge estimated by free vibration tests.

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